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Connecting Visual Objects Reduces Perceived Numerosity and Density for Sparse but not Dense Patterns

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Abstract

How is numerosity encoded by the visual system? – directly, or derived indirectly from texture density? We recently suggested that the numerosity of sparse patterns is encoded directly by dedicated mechanisms (which have been described as the "Approximate Number System" ANS). However, at high dot densities, where items become "crowded" and difficult to segregate, "texture-density" mechanisms come into play. Here we tested the importance of item segmentation on numerosity and density perception at various stimulus densities, by measuring the effect of connecting visual objects with thin lines. The results confirmed many previous studies showing that connecting items robustly reduces the apparent numerosity of patterns of moderate density. We further showed that the apparent density of moderate-density patterns is also reduced by connecting the dots. Crucially, we found that both these effects are strongly reduced at higher numerosities. Indeed for density judgments, the effect reverses, so connecting dots in dense patterns increases the apparent density (as expected from the physical characteristics). The results provide clear support for the three-regime framework of number perception, and suggest that for moderately sparse stimuli, numerosity – but not texture-density – is perceived directly.

Keywords: numerical cognition, Approximate Number System, numerosity discrimination, numerosity perception, texture-density, texture perception, visual segmentation

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Even under conditions where individual items are uncountable, humans can make rapid and reasonably accurate estimates of the number of items in a scene, as can young infants and many animal species (Dehaene, 1997; Nieder, 2005, 2013; Whalen, Gallistel, & Gelman, 1999; Xu, Spelke, & Goddard, 2005). Estimation of the number of items extends over a wide range, from a few units to hundreds of items. Much evidence shows that numerical estimation is subserved by two separate systems: one fast and errorless, handling very few items (usually up to four) termed *subitizing* (Jevons, 1871; Kaufman & Lord, 1949); the other slower and error-prone, estimating higher numerosities, often termed the *Approximate Number System* (Butterworth, 2010; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992).

We have recently suggested a further division, between *estimation* (by the ANS) for moderate numerosities, higher than the subitizing range but not too dense to segregate items, and texture density, when objects cannot be segregated (Anobile, Cicchini, & Burr, 2016). Over the estimation range discrimination thresholds follow Weber's law, remaining proportional to numerosity (a commonly accepted signature of the Approximate Number System), while at higher numerosities thresholds follow a square-root law, as predicted by their signal-to-noise ratios. These results suggest that within a certain range, when the items are sparse enough to permit spatial segregation, numerosity is encoded directly by the Approximate Number System, without involving texture-density mechanisms (Anobile, Cicchini, & Burr, 2014; Anobile, Cicchini, et al., 2016; Anobile, Turi, Cicchini, & Burr, 2015). When the ensembles become too crowded, another system seems to come into play, encoding texture-density.

Considerable evidence has accumulated to reinforce the idea that number is encoded directly at moderate densities. For example, Stoianov and Zorzi (2012) have shown that number (rather than density) emerges spontaneously within an unsupervised learning algorithm. If numerosity were a sensory by-product of density and area, as suggested by several research groups (Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Durgin, 2008; Morgan, Raphael, Tibber, & Dakin, 2014; Tibber, Greenwood, & Dakin, 2012; Tibber et al., 2013), sensitivity for numerosity should be predictable from the independent measures of area and density. We recently directly tested this hypothesis and showed that when subjects discriminate items varying in the space spanned by number, density and area, discrimination thresholds are far lower for number than for density of area, suggesting that number rather than density is sensed spontaneously (Cicchini, Anobile, & Burr, 2016). When subjects were specifically asked to make numerosity, density and area judgments in a range of sparse stimuli varying in all three dimensions, all three dimensions interacted with each other. However, number had a much stronger effect on density and area than vice versa. Density judgements were biased towards numerosity by about 78% and area by 53%, whereas number judgements were biased towards area by only 15%. This is consistent with several studies that showed small biases of numerosity estimates towards area (DeWind, Adams, Platt, & Brannon, 2015; Gebuis & Reynvoet, 2012a, 2012b), but stronger effects of numerosity on area judgements (Hurewitz, Gelman, & Schnitzer, 2006; Nys & Content, 2012) and density judgments (Dakin et al., 2011). Importantly, the selective sensitivity for numerosity over density is far less pronounced with dense stimuli, where the results are consistent with independent analyses of density and area (Cicchini et al., 2016). This is supported by several other studies showing clear differences in the psychophysical laws governing visibility at high and low densities of dot patterns (Anobile et al., 2014; Anobile et al., 2015; Cicchini et al., 2016), leading to the suggestion of two different regimes of analysis (Anobile, Cicchini, et al., 2016).

A particularly clear demonstration that numerosity mechanisms need not involve texture density is the cross-modal and cross-format adaptation study of Arrighi, Togoli, and Burr (2014). Adaptation is one of the clearest signatures of the existence of a dedicated system for the encoding of a particular feature (Clifford & Rhodes, 2005; Mollon, 1974; Thompson & Burr, 2009). This widely used method involves the quantitative measurement of the perceptual distortion caused by the previous exposure to a given stimulus. For example, inspecting for a few seconds the downward motion of a waterfall (adapter stimulus), the surrounding rocks will be perceived as moving upwards. This is the signature of a neural system dedicated to those motion directions. Numerosity is also susceptible to adaptation: viewing large numbers of dots for a few seconds (adapter stimulus) makes subsequent smaller groups of dots seem to contain fewer dots than they actually do (Burr, Anobile, & Turi, 2011; Burr & Ross, 2008; Schwiedrzik, Bernstein, & Melloni, 2016). Recently, Arrighi et al. (2014) extended the technique to demonstrate adaptation to temporal numerosity sequences: adapting to a sequence of tones for few seconds changed the apparent numerosity of subsequent streams of visual flashes, and vice-versa, even though no texture was involved (as stimuli were temporarily defined). Importantly, adapting to a series of flashes changes the apparent numerosity of dot arrays: again, there was no texture in the adaptor, yet a spatial pattern seemed to contain fewer dots. They have further extended the technique to show that action adaptation can change numerosity. After tapping for a period of time (either rapidly or slowly), the apparent numerosity of a sequence of flashes and of a dot array was strongly changed, in the opposite direction to the tapping speed (Anobile, Arrighi, Togoli, & Burr, 2016). All these adaptation effects are difficult to reconcile with the notion that numerosity is a surrogate of texture-density.

However, the idea that numerosity could be perceived directly is not uniformly accepted. Many authors still maintain that numerosity is encoded via texture-density mechanisms, operating on the global statistics of images (Dakin et al., 2011; Durgin, 2008; Morgan et al., 2014; Tibber et al., 2012; Tibber et al., 2013). In particular, Dakin et al. (2011) have proposed a theory where numerosity is not sensed directly, but calculated from the product of density and area. Their model suggests that numerosity can be derived from the power spectrum of the stimulus, essentially the magnitude of high spatial frequencies normalised by the low frequency content. This theory makes very clear and testable predictions: adding visual items to the scene, especially those having greater energy in the higher spatial frequencies, should make the stimulus as more dense – and hence more numerous. In this study we specifically test this prediction.

One important feature of numerical estimation is that it is relatively invariant to the shape of the objects (Dehaene & Changeux, 1993). A consequence of this is that if two items are connected, simply by joining them with lines, the apparent numerosity of the connected items is greatly reduced (Franconeri, Bemis, & Alvarez, 2009; He, Zhang, Zhou, & Chen, 2009; He, Zhou, Zhou, He, & Chen, 2015) – see examples of Figure 1, and also the Supplementary Movie. In fact the lines do not need to be physically present: the effect works well with illusory contours as well (Kirjakovski & Matsumoto, 2016). Connecting dots not only changes the perceived numerosity of a pattern, but also the selectivity of the fMRI BOLD response. Connecting three dot-pairs in a pattern of ten dots causes the maximum repetition adaptation to occur at eight rather than ten dots (He et al., 2015). Similarly, while adapting to a 20-dot field does not change the apparent numerosity of a 20 isolated dots (as adaptation to the same number has no net effect), it does reduce the apparent numerosity of 10 dot-pairs, which appear less numerous than 10 isolated dots, and even less numerous after adaptation (Fornaciai, Cicchini, & Burr, 2016). These studies are particularly interesting in the context of the numerosity texture-density debate: adding connectors increases the amount of texture on the screen, particularly at high spatial frequencies (see Figure 1), yet people underestimate numerosity, rather than overestimate it, as predicted by global texture-based models (Dakin et al., 2011).

In this study we ask whether the connectedness effect also occurs with dense pattern, where we suggest that texture mechanisms come into play. We predicted that the effect of connectivity on numerosity should be strongly reduced at high numerosities, where the Approximate Number System gives way to texture-density mechanisms. We also predicted that for dense patterns, where texture-density mechanisms predominate, subjects should see the connected patterns as more dense than the isolated patterns: the reverse would hold true for sparse patterns. We therefore measured both apparent numerosity and apparent density in dot clouds with items connected, over a range of numerosities.

Methods

Participants

Five subjects (1 male, 4 females, all naïve to the goals of the experiment, mean age 25 years) took part in the study. All had normal or corrected-to-normal visual acuity and no major visual impairment. All participants gave written informed consent. Experimental procedures were approved by the local ethics committee (*Comitato Etico Pediatrico Regionale—Azienda Ospedaliero-Universitaria Meyer*, Florence), and are in line with the declaration of Helsinki.

Stimuli and Procedure

Both numerosity and density thresholds were measured with a classical two alternative forced choice method (2AFC): participants were asked to report (by appropriate keypress) which of two stimuli (clouds of dots, diameter 6.2 degrees) seemed to be more numerous or more dense, guessing whether uncertain. Stimuli were presented simultaneously on both sides of a central fixation point (8 degrees eccentricity), too fast to make single elements serially countable (500 ms).

Subjects sat at 57 cm from a calibrated LCD screen (35 x 19 degrees) running at 60 Hz and 1366 X 768 resolution (1 pixel = 0.025 degrees). Stimuli were generated under Matlab 7.6 using PsychToolbox routines (Kleiner, Brainard, & Pelli, 2007). One of the two dot clouds, the *reference*, remained of fixed numerosity throughout the session, while the *probe* varied in numerosity throughout the session, guided by the adaptive Quest routine (Watson & Pelli, 1983) which homed in on the Point of Subjective Equality (PSE). The probe comprised only isolated dots, whereas the reference comprised either isolated dots, or patterns with 40% dots connected by joined by lines: Figure 1 (B and E). In separate runs the experiment was performed with reference stimuli of 15, 25, 50 and 100 items, corresponding to densities of 0.5, 0.83, 1.67 and 3.3 items/deg^2. For density judgements, to maximize information while keeping the number of conditions reasonably low, only the two extreme values were tested (0.5 and 3.3 items/deg^2). Each participant performed 100 trials for each numerosity/density condition, for both connected and isolated dots resulting in a total of 1200 trials for each participant (6000 trials across participants and conditions, half trials connected and half isolated).

Dots were small disks of 0.25 degrees diameter, half white, half black (so that luminance did not vary with number, providing a potential cue). Dot position was calculated online for each trial. For patches containing isolated dots, dot positions were generated sequentially, respecting the sole condition that two items could not be closers than 0.25 degrees (10 pixels) thus forbidding dots overlap. For patches with connectors, dot position was calculated in two stages: first couples of dots (40% of the total dots of the reference stimulus) were cast and connected via a line, with the constraints that line length was comprised between 1 and 1.5 degrees, with no lines crossing; in the second stage, the remaining 60% of the dots were cast with the constraint of not overlapping either the other dots or the connecting lines. The connector line width was fixed at 0.05 degrees (2 pixels). Figure 1 shows examples of the patterns used in the experiments.

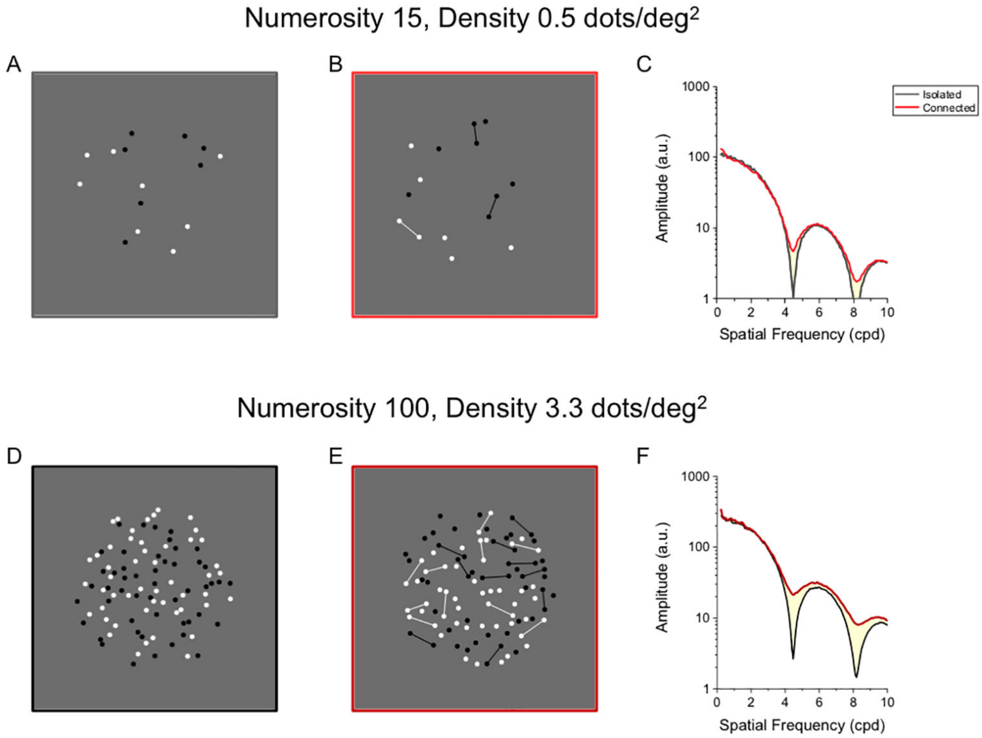

Figure 1. Illustration of stimuli and their Fourier transforms. Top row: Stimuli at low numerosity/density. (A) Sample patch containing only isolated dots (Numerosity = 15, Density = 0.5 dots/deg^2) (B) Stimulus of same numerosity but with 6 dots (40%) connected by thin lines. (C) Radial Fourier Spectra of the two patches: isolated dots (black), connectors (red). Note that the connected patterns have more energy at relatively high spatial frequencies, 4-6 c/deg. Bottom row (D-F): Stimuli at high numerosity/density (Numerosity = 100, Density = 3.3 dots/deg^2). Other conventions as above.

Data Analysis

Data were analysed separately for each subject. For each condition (numerosity or density judgments, with connected or isolated items) and reference numerosity, the responses were plotted as function of the probe numerosity on a logarithmic scale and fitted with a cumulative Gaussian distribution, whose median estimates the Point of Subjective Equality (PSE) (see Figure 2). In our paradigm the connected dots were always the reference: underestimation of the reference results in a PSE below the true numerosity of the patch (see Figure 2). The effect of connectivity was defined as:

$$connectivity = \frac{PSE_{connected}}{PSE_{isolated}} \times 100 \tag{1}$$

The effect of item connection and numerosity were analysed by two-way repeated measures ANOVA, conducted with Sigmaplot 12 for Windows (Systat Software, Inc, California, USA).

Results

Figure 1 shows examples of the patterns used in the experiments, both high and low densities with connected and unconnected dots. It is clear from inspection that connecting three dot-pairs (20%) of the low-density pattern (Figure 1B) visibly reduces apparent numerosity, while the effect of connecting 20 dot-pairs at high

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densities (again 20%: Figure 1E) is far less obvious. The figures at right show the amplitude spectra of the patterns: isolated dots in black, connected in red. The amplitude spectra were calculated by Fourier analysis, which essentially decomposes the images into a series of sinusoidal waveforms, of different spatial frequencies, orientations and amplitudes. This analysis yielded two-dimensional amplitude spectra, which were averaged over all orientations to yield the one-dimensional amplitude plots of Figures 1C and 1F. At both densities the spectra are similar: amplitudes decrease with spatial frequency, similarly to most natural images (Field, 1989; Tolhurst, Tadmor, & Chao, 1992). There is also a clear dip in amplitude around 4 c/deg, largely driven by the size of the dots (0.3 deg diameter). Importantly, the spectra for the connected dots have higher amplitude over this high frequency range (4-6 c/deg). According to the energy-based models of Dakin and collaborators (e.g. Dakin et al., 2011; Morgan et al., 2014), these patterns should appear more numerous than those with isolated dots.

Figure 2 shows psychometric functions for the tasks. These example functions plot *aggregate data,* pooling over all subjects to illustrate the technique: but all subsequent analysis was done with similar functions for individual subjects. The upper panels (A and B) show data for 15-dot stimuli (0.5 dots/deg^2), bottom panels (C and D) for 100-dot stimuli (3.33 dots/deg^2). For the sparse patterns, the PSEs for numerosity judgements are clearly shifted to the left for the connected stimuli, implying that they seem to contain fewer elements, agreeing with all the previous literature (Fornaciai et al., 2016; Franconeri et al., 2009; He et al., 2009; He et al., 2015; Kirjakovski & Matsumoto, 2016). For the dense pattern there is still a difference in PSEs, but the difference is much reduced, from about 40% for 15 dots to about 17% for 100 dots.

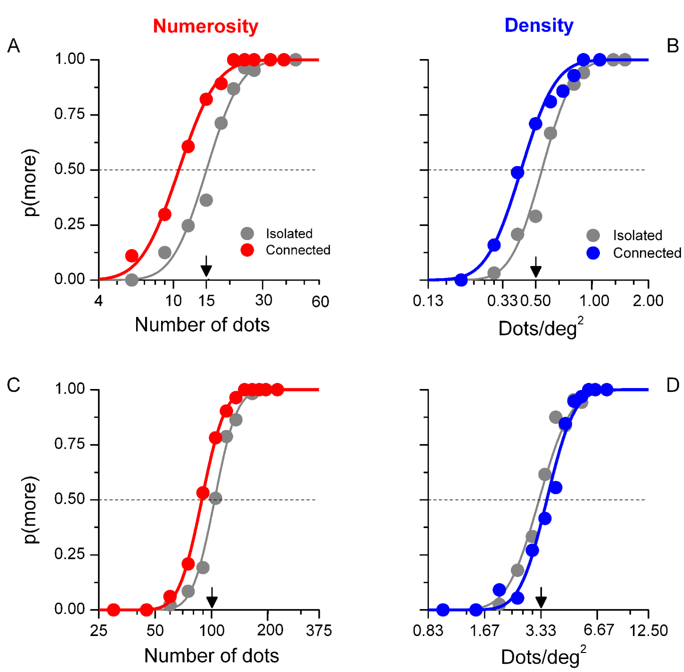

Figure 2. Aggregate psychometric functions for low and high numerosities: A and B 15 dots (density 0.5 dots/deg^2, C and D 100 dots (density 3.3 dots/deg^2). A and C show judgements of numerosity, B and D density (in separate sessions). All graphs plot the proportion of times the subject reported the probe to be more numerous (or more dense) than the reference, as a function of probe number. Coloured lines refer to the connected condition (reference containing 40% of connected dots), greys lines to the isolated dots condition (baseline). Leftward shifts of coloured curves imply underestimation of the connected-dot stimulus.

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In separate sessions we also asked subjects to judge the apparent density of the patches. The results for the aggregate observer are shown on the right panels of Figure 2 (B and D). Despite the fact that the connected patterns contain more “stuff”, particularly at high spatial frequencies, shown clearly by their Fourier transforms (Figure 1), participants under-estimated rather than overestimated the density of the 15-dot pattern when dots were connected, by about 30%. However, they overestimated the density of the 100-dot pattern, as predicted by the power spectrum.

The main analysis was performed on the data of individual participants. For each participant and condition we fitted psychometric functions like those of Figure 2, from which we extracted estimates of PSE for the various conditions. We then calculated the effect of connectivity from the PSEs for the connected and isolated dots (Equation 1). Figure 3 shows the results averaged over subjects as a function of numerosity and density. Figures 3A and 3B report the biases for the connected and isolated conditions, Figure 3C the connectivity effect (Equation 1). In all cases, the data were obtained from averaging the PSEs of individual subjects, rather than from the aggregate subject. For both numerosity and density judgements, the isolated condition had almost no bias. And for both judgements, the bias of the connected stimuli decreased with numerosity. For numerosity judgments the bias remained negative for 100-dot stimuli, while for density the effect crosses over, so the connected patterns appear denser. These clear effects result in statistically significant interactions between numerosity and biases (two-way repeated measures ANOVA: $F_{(3,12)}$ = 4.049, p = 0.033 and $F_{(1,4)}$ = 44.548, p = 0.003 for numerosity and density tasks respectively).

Figure 3C shows connectivity effect (difference between isolated and connected conditions) for the two tasks. Two way repeated measures ANOVA with factors Dot Number and Task revealed a significant main effect of dot number ($F_{(1,4)}$ = 52.1 p = 0.002) consistent with the idea that the connectivity effect is modulated by stimulus density (number of dots).

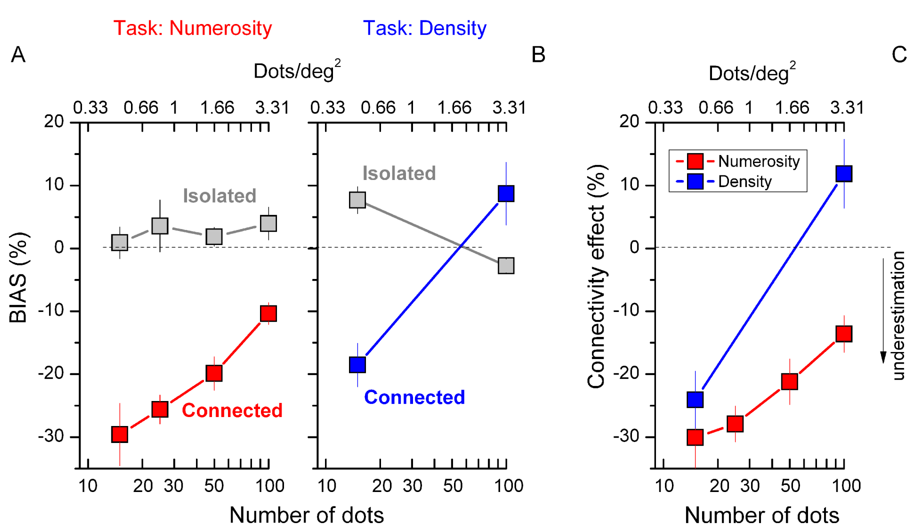

Figure 3. A and B) Average PSEs expressed as percentage difference from the references number/density levels (on the abscissae). The isolated dots condition (baseline) is reported in grey. Numerosity judgements for connected stimuli are shown in red, density judgments in blue. C) Effect of connecting dots expressed as % difference from PSEs in the isolated and connected conditions (colour code as above).

Figure 4 shows the results for individual subjects, plotting the connectivity effect for high (100-dot) against low (15-dot) patterns, for both numerosity and density. The pattern of results is very consistent amongst subjects. All points fall to the left of zero, meaning that both numerosity and density PSEs of the 15-dot patterns were underestimated by all five subjects. All points also clearly fall above the equality line (dashed), meaning that both the numerosity and density effects were stronger at low than at high numerosities. The effects were clearest for the densities judgments: all subjects judged the connected pattern to be less dense for 15-dot patterns, but more dense for 100-dot patterns (in the upper left-hand quadrant).

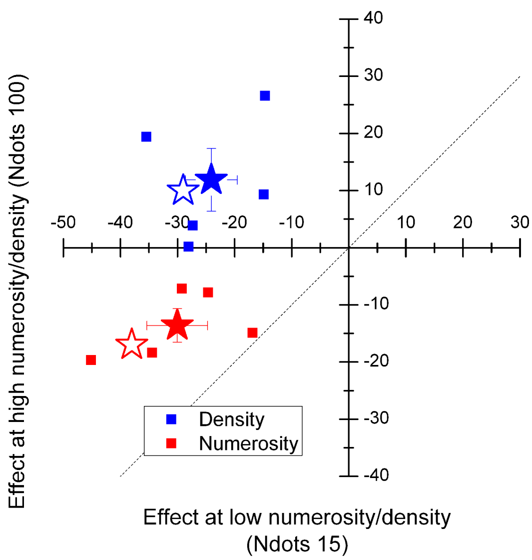

Figure 4. Effect of connectivity (normalized percentage distance between PSEs of isolated and connected stimuli: Equation 1 for numerosity (red) and density (blue) judgements. The effect for 100-dot stimuli is plotted on the ordinate against that for 15-dot stimuli (on the abscissa). Squares report single subject data, filled stars the between subjects average effects, open stars the effect calculated on the aggregated subject.

Discussion

We studied the effect of stimulus connectivity on numerosity and density judgments in adult subjects, for various dot densities. We confirmed previous studies showing that connecting items in the visual scene results in a strong and reliable underestimation of numerosity (Fornaciai et al., 2016; Franconeri et al., 2009; He et al., 2009; He et al., 2015). We expanded these results to show that: 1) this effect is strongly reduced at higher numerosities; 2) when asked to judge density rather than numerosity, participants underestimated the *density* as well as the numerosity of stimuli with connectors for sparse patterns; 3) as for numerosity, the effect of connectivity on apparent density depends on item spacing: for sparse items, density was underestimated, switching to overestimation for denser stimuli.

The results confirm the predictions that number is directly sensed only if the density is low enough to permit segregation: for highly packed stimuli, the Approximate Number System – which operates on segregated items – seems to be less active, so estimates were less influenced by the manipulation affecting segregation. However, for sparse stimuli, numerosity mechanisms dominate, and estimates followed what would be expected if numerosity were based on segregable units, with connected items perceptually merged.

It is particularly interesting that texture-density estimates for sparse stimuli did not follow the overall energy in the stimulus, which is higher for connected patches, particularly at high spatial frequencies (Figures 1C and 1F), but was also affected by item-connectivity. As mentioned in the introduction, the influential theory (Dakin et al., 2011) suggesting that numerosity could be calculated from the power spectrum of the stimulus makes the prediction that connected items, which have greater energy in the higher spatial frequencies, should appear more dense – and hence more numerous. Indeed this prediction is upheld for dense patterns, at least for density judgments: but for low and modest densities it fails completely. This result not only shows that numerosity is not derived indirectly from density, texture or other low-level features, but also casts doubts on the idea that density is a primary visual feature: rather, density seems to be derived indirectly from numerosity, reinforcing previous evidence showing that at low numerosities, density judgments are particularly unreliable and are often surrogated by number judgments (Cicchini et al., 2016). Overall our results add to a growing body of literature showing that visual numerosity is perceived directly, rather than being recalculated from area and density (Anobile, Arrighi, et al., 2016; Anobile, Castaldi, Turi, Tinelli, & Burr, 2016; Anobile et al., 2014; Anobile, Cicchini, et al., 2016; Anobile et al., 2015; Arrighi et al., 2014; Burr & Ross, 2008; Cicchini et al., 2016; DeWind et al., 2015; Hurewitz et al., 2006; Kramer, Di Bono, & Zorzi, 2011; Ross & Burr, 2010, 2012; Stoianov & Zorzi, 2012).

Understanding the perceptual factors limiting the Approximate Number System (ANS) is also important because of the possible role of the ANS as the evolutionary non-symbolic basis from which symbolic mathematical knowledge is built (Piazza, 2010). Evidence for this idea includes the fact that ANS acuity of typical school- and preschool-aged children have been found positively correlated with their actual and future math skills (Anobile, Stievano, & Burr, 2013; Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011, 2013; Pinheiro-Chagas et al., 2014; Starr, Libertus, & Brannon, 2013) – but see also (Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, Defever, Maertens, & Reynvoet, 2014). Furthermore, dyscalculic children have often been found to have poorer numerosity thresholds (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010) – but see also Rousselle and Noel (2007) for a different account. Besides, the investigation of numerosity perception is expanding to other developmental disorders like autism (Turi et al., 2015), motor coordination disorders (Gomez et al., 2015) and preterm children (Hellgren, Halberda, Forsman, Ådén, & Libertus, 2013; Tinelli et al., 2015).

As understanding numerosity mechanisms has important implications for understanding the development of math skills, it is important to be clear over what range we expect these mechanisms to operate. We recently showed that numerosity discrimination thresholds for sparse but not for dense dot arrays correlated with formal math skills in a group of primary school children (Anobile, Castaldi, et al., 2016). These results are also in line with the adult literature, where numerosity but not texture-density discrimination thresholds correlated with math abilities (Tibber et al., 2013).

Conclusions

Does the Approximate Number System operate on unsegmented visual scenes, as suggested by Durgin (2008) and Dakin et al. (2011) or does it operate on segmented objects (Anobile, Cicchini, et al., 2016; Castelli, Glaser, & Butterworth, 2006; Fornaciai et al., 2016; Franconeri et al., 2009; He et al., 2009; He et al., 2015; Kirjakovski & Matsumoto, 2016)? This study strengthens the idea that the Approximate Number System operates only when items are sparse enough to permit spatial segregation. After that limit, ANS gives way to another separate perceptual system responding to texture-density. We also suggest that for sparse stimuli, numerosity – but not element density – can be directly perceived, without being calculated indirectly from other perceptual features.

Supplementary Materials

Supplementary Movie

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Competing Interests

The authors have declared that no competing interests exist.

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Author Contributions

Giovanni Anobile and Guido Marco Cicchini contributed equally to this work.

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