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Spatial but Not Temporal Numerosity Thresholds Correlate With Formal Math Skills in Children

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Humans and other animals are able to make rough estimations of quantities using what has been termed the *approximate number system* (ANS). Much evidence suggests that sensitivity to numerosity correlates with symbolic math capacity, leading to the suggestion that the ANS may serve as a start-up tool to develop symbolic math. Many experiments have demonstrated that numerosity perception transcends the sensory modality of stimuli and their presentation format (sequential or simultaneous), but it remains an open question whether the relationship between numerosity and math generalizes over stimulus format and modality. Here we measured precision for estimating the numerosity of clouds of dots and sequences of flashes or clicks, as well as for paired comparisons of the numerosity of clouds of dots. Our results show that in children, formal math abilities correlate positively with sensitivity for estimation and paired-comparisons of the numerosity of visual arrays of dots. However, precision of numerosity estimation for sequences of flashes or sounds did not correlate with math, although sensitivities in all estimations tasks (for sequential or simultaneous stimuli) were strongly correlated with each other. In adults, we found no significant correlations between math scores and sensitivity to any of the psychophysical tasks. Taken together these results support the existence of a generalized number sense, and go on to demonstrate an intrinsic link between mathematics and perception of spatial, but not temporal numerosity.

Keywords: numbr sense, numerosity perception, numerical cognition, developmental dyscalculia, approximate number system

Humans and many other animals can make rapid but approximate estimates of nonsymbolic numerical magnitudes (numerosity). This nonverbal ability is often referred to as the *number sense* or *approximate number system* (ANS; Dehaene, 2011). The sensory precision of this system refines during development, and varies considerably between individuals (Halberda, Mazzocco, & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Odic, Libertus, Feigenson, & Halberda, 2013). Importantly, strong correlations have been found between precision in numerosity judgments and formal math abilities (Anobile, Stievano, &

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Burr, 2013; Chen & Li, 2014; Cicchini, Anobile, & Burr, 2016; Feigenson, Libertus, & Halberda, 2013; Halberda et al., 2008; Libertus, Odic, Feigenson, & Halberda, 2016; Piazza, 2010; Starr, Libertus, & Brannon, 2013), leading to the suggestion that the ANS might act as a primitive "start-up" tool for subsequent math acquisition (Piazza, 2010).

There is evidence that training on nonsymbolic approximate number tasks yields improvements in symbolic arithmetic performance in adults (Park & Brannon, 2013, 2014), school-age children (Hyde, Khanum, & Spelke, 2014; Rasanen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Dehaene, Dubois, & Fayol, 2009; Wilson et al., 2006; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006) and preschoolers (Park, Bermudez, Roberts, & Brannon, 2016). Training effects have also been reported in the opposite direction, as math formal education was found to significantly enhance precision in numerosity estimation tasks (Piazza, Pica, Izard, Spelke, & Dehaene, 2013).

Although the above mentioned studies strongly support a causal link between ANS and math capacity, the issue remains controversial (Lindskog & Winman, 2016). For example, several groups have failed to find a correlation between ANS precision and math abilities (Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Rousselle & Noël, 2007; Sasanguie, Defever, Maertens, & Reynvoet, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012), while others found that training on approximate numerosity does not change formal math abilities (Obersteiner, Reiss, & Ufer, 2013; Sullivan, Frank, & Barner, 2016). Indeed, the very notion of a "sense of number" has itself been challenged (Durgin, 2008; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Leibovich, Katzin, Harel, & Henik, 2017), leading to much debate (Anobile, Cicchini, & Burr, 2016; Cicchini et al., 2016). Given the controversial nature of the evidence, and the possible clinical implications of understanding the relationship between ANS and math learning (such as improving treatments of dyscalculia), it is important to understand well the relationship between numerosity and math skills.

Numerosity is not restricted to spatial ensembles of visual stimuli. Sequences of sounds and also visual events can be easily numerated. Recent adaptation studies have provided strong evidence for the existence of a generalized number system, encoding numerosity across space and time, and across different senses. Adapting to a fast sequence of numerosities causes underestimation of subsequent stimuli, while adapting to a slow sequence causes overestimations (Arrighi, Togoli, & Burr, 2014). The effects generalize from audition to vision and vice versa, and also from sequences of flashes to spatial arrays. Numerosity adaptation even generalizes between actions and vision: adapting to midair finger-tapping (without sensory feedback) distorts the perceived numerosity of sequences of flashes, and also spatial arrays (Anobile, Arrighi, Togoli, & Burr, 2016). Other studies have shown that preschoolers (Barth, La Mont, Lipton, & Spelke, 2005), as well as adults (Barth, Kanwisher, & Spelke, 2003; Brannon, 2003), efficiently perform cross-modal and cross-format judgments, with little cost in either accuracy or reaction times (RTs) when comparing auditory with visual temporal sequences or dot arrays. Infants (Jordan & Brannon, 2006) and newborns (Izard, Sann, Spelke, & Streri, 2009) preferentially look at ensembles of visual stimuli (faces or abstract shapes) numerically matched with ongoing auditory stimuli (soundtrack of adult voices or sequences of sounds). These studies suggest that soon after birth, humans may be equipped with an already functional "core knowledge system," able to spontaneously focus on nonsymbolic quantities, independently of stimuli format and modality (Dehaene & Brannon, 2011; Dehaene, Izard, Pica, & Spelke, 2006; Dillon, Huang, & Spelke, 2013; Feigenson, Dehaene, & Spelke, 2004). This language-independent system may be the evolutionary base for abstract math.

Given that judgments of simultaneous and sequential numerosity are likely to share some perceptual mechanisms, and that simultaneous numerosity sensitivity predicts math skills, it is reasonable to ask whether sensitivity in sequential numerosity is also a good predictor of math. Does the sensory modality of the stimuli play a role in the link between ANS and math skills? Similarly, are both discrimination and estimation processes of numerosity good predictors of math? And finally, do math abilities correlate unspecifically with higher sensitivity in nonnumerical quantity perceptual tasks?

We measured ANS precision in children and adults on several psychophysical tasks: verbal magnitude estimation ("how many?") of dot ensembles, series of flashes or streams of sounds, and nonverbal discrimination ("which pattern has more?") of simultaneous numerosity (dot ensembles) and disk size. Math abilities were measured by assessing performance on mental calculation and tasks measuring numerical magnitude knowledge ("select the highest digit"; "place the target number in the appropriate position"). The hypotheses are straightforward: if the cognitive systems sustaining mathematical cognition encode simultaneous as well as sequential numerosity, we expect to find significant correlations in all cases; on the other hand, correlations in one domain and not the other would suggest the existence of at least partially different mechanisms.

Method

Participants

One hundred forty-four subjects participated: 105 children (7–11 years old, $M = 9.0$), and 39 adults (19–30 years, $M = 25.7$). Children were recruited from local schools, and only those who returned a signed consent from parents were included. Experimental procedures were approved by the local ethics committee (*Comitato Etico Pediatrico Regionale—Azienda Ospedaliero-Universitaria Meyer*—Florence, Italy; project: Early Sensory Cortex Plasticity and Adaptability in Human Adults) and are in line with the Declaration of Helsinki.

General Procedures

Stimuli were generated and presented with MATLAB 8.1 using PsychToolbox routines (Brainard, 1997) on a 17-in. LG touch screen AQ: 3 monitor with 1,280 × 1,024 resolution at refresh rate of 60 Hz (model number: L1730SF, Milano, Italy). Each participant was tested in two separate sessions (usually occurring within the same week), lasting around 1 hr each. Math abilities were measured by a paper-and-pencil test (only children) and by a computerized digit summation task. All participants also performed a nonverbal reasoning task (Raven matrices). Math skills and nonverbal reasoning were usually measured at the end of the first session.

Numerosity Discrimination

Two patches of dots were briefly (250 ms) presented simultaneously on either side of central fixation. Participants indicated the side of the screen with more dots. The numerosity of the test stimulus (randomly left or right) was 24, while the probe adaptively changed following a QUEST (Quick Estimation by Sequential Testing) algorithm (Watson & Pelli, 1983). Dots were 0.25° diameter, half white and half black, presented at 80% contrast on a gray background of 40 cd/m^2. They were constrained to fall within a virtual circle of 10° diameter, centered at 8° eccentricity. All participants performed two sessions of 35 trials. The proportion of "test greater" trials was plotted against the log ratio of test and probe, and fitted with cumulative F1 Gaussian error functions (Figure 1A). The 50% point of the error functions estimates the point of subjective equality (PSE), and the difference in numerosity between the 50% and 75% points gives the just notable difference (JND), which was used to estimate Weber fractions (WFs; JND/PSE). Note that JND and WFs are both estimates of thresholds, the inverse of sensitivity.

Numerosity Magnitude Estimation

Visual stimuli were either ensembles of dots (diameter 0.5°, half white and half black), presented simultaneously for 250 ms within a virtual 16° diameter region, or sequences of flashes (sharp-edged white disks of 90 cd m^{-2} and 5° diameter) presented in a pseudorandom order within a 2-s interval. In the sequential presentation, each flash lasted 40 ms with the constraint that two pulses could not fall within 40 ms of each other. All visual stimuli were presented centrally, with subject viewing distance set at 57 cm, on a gray background of 40 cd/m^2. Precision for numerosity estimates of sequential stimuli was also investigated in audition, with 500 Hz pure tones ramped on and off with 5-ms raised-cosine ramps, presented with an intensity of 80 dB (at the sound source) and digitized at a rate of 65 kHz. Sounds were presented through high-quality headphones Microsoft lifechat LX-3000 (Microsoft, Redmond, Washington), and AQ: 5 perceptually localized in the middle of the head. In all conditions the numerosity range was 2–18, and subjects asked to verbally report the number of perceived stimuli, which the experimenter recorded via computer keyboard. The testing phase was preceded by a training session of 17 trials (not included in the main analyses). During training, all numerosities were randomly presented, and feedback provided by displaying the actual numerosity displayed on the monitor screen. After training had been completed, the testing phase started with a block of 51 trials (three repetitions for each numerosity), with no feedback. In total each participant performed 204 trials. Test numerosity ranged from 2 to 18, but we analyzed only the range 5–16 to avoid the subitizing range as well as edge effects (e.g., from subjects knowing or guessing that the numerosity never exceeded 18 dots). Precision was defined as the WF, the standard deviation of response distributions normalized by the average response; WFs were averaged across all numerosities.

Size Discrimination

Stimuli were gratings sinusoidally modulated in luminance with a spatial frequency of 2 cycles per degree, a Michelson contrast of

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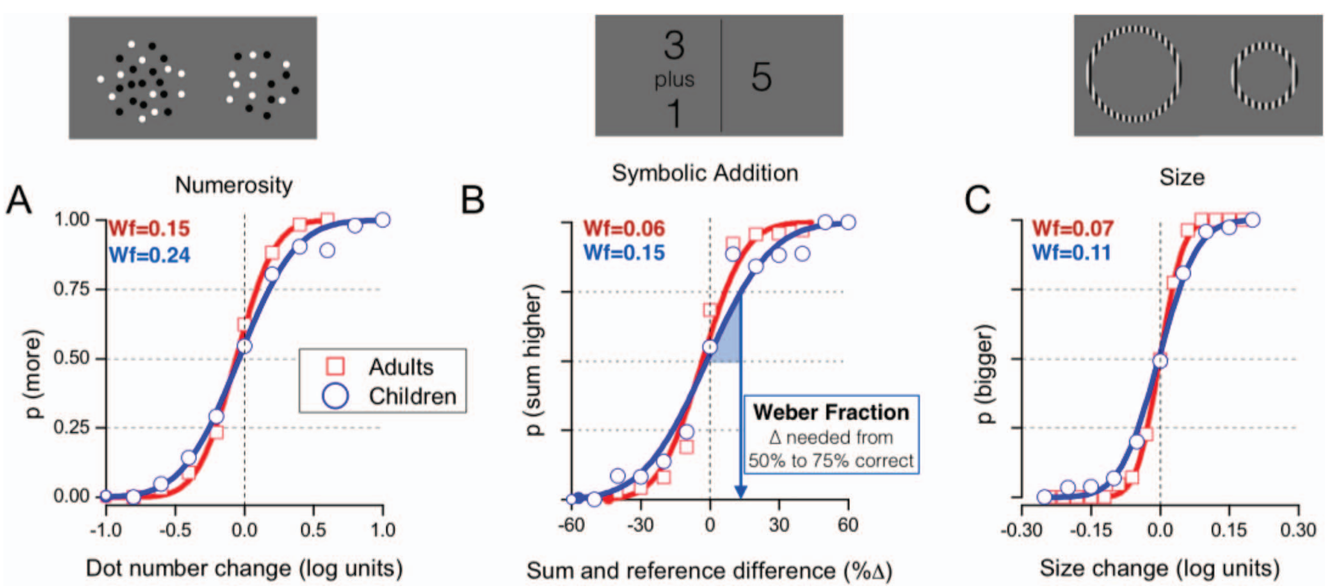

Figure 1. Psychophysical discrimination tasks. Aggregate psychometric functions for children (blue circles and lines) and adults (red squares and lines) for three different discrimination tasks. (A) Numerosity discrimination: Participants were required to select which one out of two briefly presented (250 ms) dots ensemble was more numerous. (B) Symbolic addition discrimination: On each trial, participants were asked to mentally add—as quickly as possible—the digits numbers on the left and compare the sum with that on the right (5 in this example), indicating which side was numerically higher (right in the example: 3 + 1 vs. 5). Stimuli remained until response. (C) Size discrimination: Participants were asked to indicate which of two briefly (250 ms) presented annulus was perceived as larger (method adapted from Pooresmaeili, Arrighi, Biagi, & Morrone, 2013). In all cases, discrimination precision was measured by WF (just notable difference/point of subjective equality). For example, a WF of 0.15 in the symbolic addition task (B) indicates that the sum of the two addenda had to be 15% higher or lower than reference to raise responses from chance to 75% correct responses. See the online article for the color version of this figure.

90% that were vignetted by annular contrast window (see insert to Figure 1C). In each trial two annuli were simultaneously presented for 250 ms on the left and right side of the fixation point, centered at 10° eccentricity, and subjects required to indicate which appeared to be larger. The diameter of the *test* stimulus (randomly left or right) was 5° or 8° (35 trials each, randomized trial-by-trial), while the *probe* varied in diameter by a percentage drawn randomly from a Gaussian distribution centered at 0 with $SD = 20\%$. To minimize alternative judging strategies (such as estimating border-to-center of the screen distance), we independently jittered the horizontal eccentricity of the test and probe between 8.5° and 11.5°, and their distance from the horizontal meridian within ± 3°. After stimulus presentation, a 100 ms full screen random noise mask was displayed to cancel out possible afterimages. Proportion of "test larger" trials was plotted against the log-ratio of test to probe and fitted with cumulative Gaussian error functions (see Figure 1C). As before, the 50% point estimates the PSE, and the size change needed to move from 50% to 75% of correct responses gave the size discrimination threshold.

Mental Calculation Task

Mental calculation was measured by a custom computerized task. On each trial three digits (3°×3°, Geneva font) were displayed. Two of them (vertically aligned at a distance of 1.5°) were displayed to the left and one to the right of a central reference point (horizontal eccentricity 2°). Participants were required to mentally sum the two digits on the left and compare the result with the single digit on the right and thus to indicate (by appropriate key press) which side contained the higher magnitude. Both the addenda ranged from 1 to 9 and were randomly chosen, on each trial, with the sum of the two numbers constrained between 5 and 10 (grain of 1). The single digit (comparison sum) was determined by adding to the real sum a delta value chosen from a flat distribution ranging from ± 60% for children, and ± 40% for adults, rounding to the closest integer. Participants performed two blocks of 35 trials. To minimize strategies other than mental calculation (such as serial counting), we asked participants to respond as fast as possible, but accurately. We applied a time threshold (2 and 5 secs for adults and children respectively), with thresholds derived from preliminary data. In trials where RTs exceeded the threshold 5.6% and 1.8% for children and adults respectively, we gave an auditory feedback. Not every trial where RTs exceeded the threshold were eliminated from the analysis, as we applied a within subject cut-off: for each participant we measured the average RTs (across trials) and eliminated those higher or lower than 3 *SD*. The total number of eliminated trials was 38 (1.1%) for adults and 80 (1.4%) for children. The proportion of "sum higher" was plotted against the percentage difference between the sum and the single digit. As with the other discrimination tasks, we fitted the data with a cumulative Gaussian error functions (Figure 1B). The percentage difference needed to move from 50% to 75% correct responses provided an additional discrimination threshold. This is logically equivalent to the WF usually measured for numerosity discrimination tasks, and could be interpreted as the amount of noise present in the mental addition process. Similarly to Cicchini et al. (2016), we computed for each participant two separate z scores: one for precision (WF) and the other for response speed (RT). z Scores were measured using the mean and standard deviation of the participant grade class (from second to fifth grade). For adults we used the mean and standard deviation of the entire group. Finally, for each participant, we computed a performance-combined index averaging the two z scores.

Semantic Skills

Two types of paper-and-pencil task were administered (see Figure 2 for examples): (a) choose and mark the largest numbers F2 in a set of three (one to five digits, 36 trials) and (b) mark where a number should be placed (four possible positions among three other numbers, one to six digits, 18 trials). These tasks were extracted from an Italian standardized battery (Biancardi, Bachmann, & Nicoletti, 2016). They are thought to tap the semantic component of numeracy (Dehaene, Piazza, Pinel, & Cohen, 2003), and have been demonstrated to be good predictor of children numerosity discrimination thresholds (Anobile et al., 2013; Cicchini, Anobile, & Burr, 2016; Piazza, 2010). Again, accuracy and speed were measured (as the number of errors and time in minutes required to complete the three tasks), and z scores calculated separately for speed and accuracy, then combined by averaging (same technique exploited by Anobile et al., 2016). Cronbach's alpha on raw scores was 0.77. AQ: 6

Preprocessing

A priori power analysis (effect size: 0.35, $\alpha = .05$, one-tailed) reveals that to reach a power $(1 - \beta)$ of 0.8 a sample size of 46 was needed (36 in case of $[1 - \beta] = 0.7$). Our samples can detect a true correlation of 0.35 in 74% and 98% of cases for adults and children respectively. Effect size (bivariate r) was estimated from meta-analyses (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017). Seven children were eliminated from the dataset: five because they were absent during the second data collection session and two because they were subsequently diagnosed with neurodevelopmental disorders (one low cognitive functioning and one oppositional defiant disorder, and both unable to accomplish most of the tasks). For children, we had six missing values due to technical problems and participant unavailability: Two children did not perform the addition task, three children did not perform the size discrimination task, and one did not perform the dots numerosity estimation task. Missing values were left empty and data excluded with pairwise deletion method.

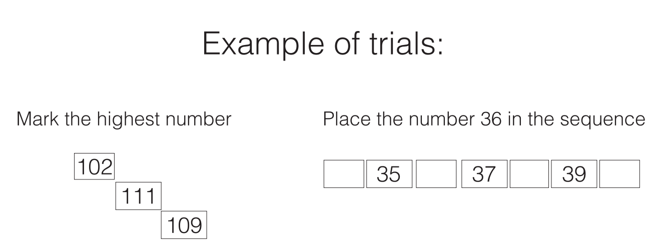

Figure 2. Example of paper and pencil math tasks. In separate blocks of trials, children were required to quickly choose and mark the highest numbers in a set of three (left panel) and to mark where a number should be placed among others (right panel). Both speed and accuracy were measured.

Data Analysis

Discrimination thresholds were separately measured for each participant and condition. The raw threshold distributions of all perceptual tasks were not normally distributed (failing the Jarque–Bera goodness-of-fit test of composite normality, but after logarithmic transform the natural scale for Weber fractions), the nonnormality became insignificant. The math scores were normal without correction. We therefore used parametric Hierarchical regression models and Pearson's correlation to search for correlations between log thresholds and math scores. However, we also used nonparametric statistics (Spearman partial ranked correlations on nontransformed data) to confirm the data trends. Separately for children and adults, we also measured and compared the reliability of the psychophysical tasks, using a split-half "sample-with-replacement" (nonparametric) bootstrap technique suitable for reliability of measures extracted form psychometric functions (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016). For the verbal numerosity tasks and the formal math tasks we also measured Cronbach's alpha. In case of split-half, for each participant we calculated two separate thresholds (or RT measurement) from a random sample of the data (as large as the data set taken, sampled with replacement from the data set), and then computed the correlation between those two measures. We reiterated the process 1,000 times for all participants, to yield mean and standard error estimates of reliability. Statistical significance was indexed by *p* values and also by the Bayes factor (Wetzels & Wagenmakers, 2012). Bayes factor is the ratio of the likelihood probabilities of the two Models H_1/H_0, where H_1 is the likelihood of a correlation between the two variables, and H_0 the likelihood that the correlation does not exist. By convention, a log Bayes factor (LBF) greater than 0.5 it is considered substantial evidence in favor of the existence of the correlation, and LBF < -0.5 substantial evidence in favor of it not existing. Absolute values of LBF greater than 1 are considered strong evidence. Values greater than ± 2 are considered decisive. Data were analyzed with both MATLAB and SPSS Version 20.0.

Results

General Results

None of the subjects, either adult or children, had any difficulty in performing any of the comparison tasks, producing orderly psychometric functions. Figure 1 shows the aggregate data for children (circles, blue) and adults (squares, red), together with their fitted psychometric functions (cumulative Gaussians). The curves are steeper for the adults than for the children, reflecting lower thresholds (higher precision) and, hence, lower WF (see Table 1 for between participants averaged values). T1

Figure 3 shows the results for the numerical magnitude estimation tasks. Figure 3A and 3B plot average responses as a function of numerosity, which were reasonably accurate (despite a slight tendency for underestimation) for all three types of stimuli (spatial arrays of dots and temporal sequences of flashes and tones). The insert in Figure 3A shows how precision (WF) was calculated for the estimation tasks: for each numerosity (in this case, 18), we divided the standard deviation of the responses by the mean estimate: the lower the precision in the estimates, the higher the standard deviation and hence the WF. The averaged WFs (across subjects) for each numerosity are reported in Figure 3C and 3D for children and adults respectively (see Table 1 for aggregate WF across numerosities). As expected, WFs were particularly low for numerosities within the subitizing range (2–4), then become constant at higher numerosities: constant WFs is one of the signatures of ANS (Anobile, Cicchini, & Burr, 2014). All our analyses focused on this estimation range so in the subsequent data analyses, numerosities within the subitizing range (≤4) were always excluded. Figure 4 shows that, as expected, precision improved with age for all numerical tasks: adults performed more precisely than children (black regression lines), and older children more precisely than younger (green regression lines). F3 F4

Table 1
Psychophysical Tasks Summary Statistics

	Children					Adults				
				Reliability					Reliability	
Tasks and stimuli	*M*	*SD*	*N*	Split-half	α	*M*	*SD*	*N*	Split-half	α
Paradigm: Discrimination										
Numerosity										
Spatial (dots)	.477	.265	98	.57 ± .12		.243	.107	38	.69 ± .12	
Size										
Disks	.098	.04	95	.68 ± .09		.057	.022	38	.66 ± .11	
Addition[a]										
Digit	.166	.14	96	Speed: .97 ± .006 Precision: .58 ± .18		.097	0.057	38	Speed: .95 ± .01 Precision: .75 ± .08	
Paradigm: Magnitude estimation										
Numerosity										
Spatial (dots)	.166	.074	97	.75 ± .06	.82	.117	.045	38	.85 ± .05	.75
Flashes	.179	.07	98	.76 ± .05	.84	.136	.05	38	.80 ± .07	.66
Sounds	.176	.077	98	.74 ± .05	.83	.117	.046	38	.85 ± .04	.83

AQ: 14 [a] Mean reaction times reported in the method section.

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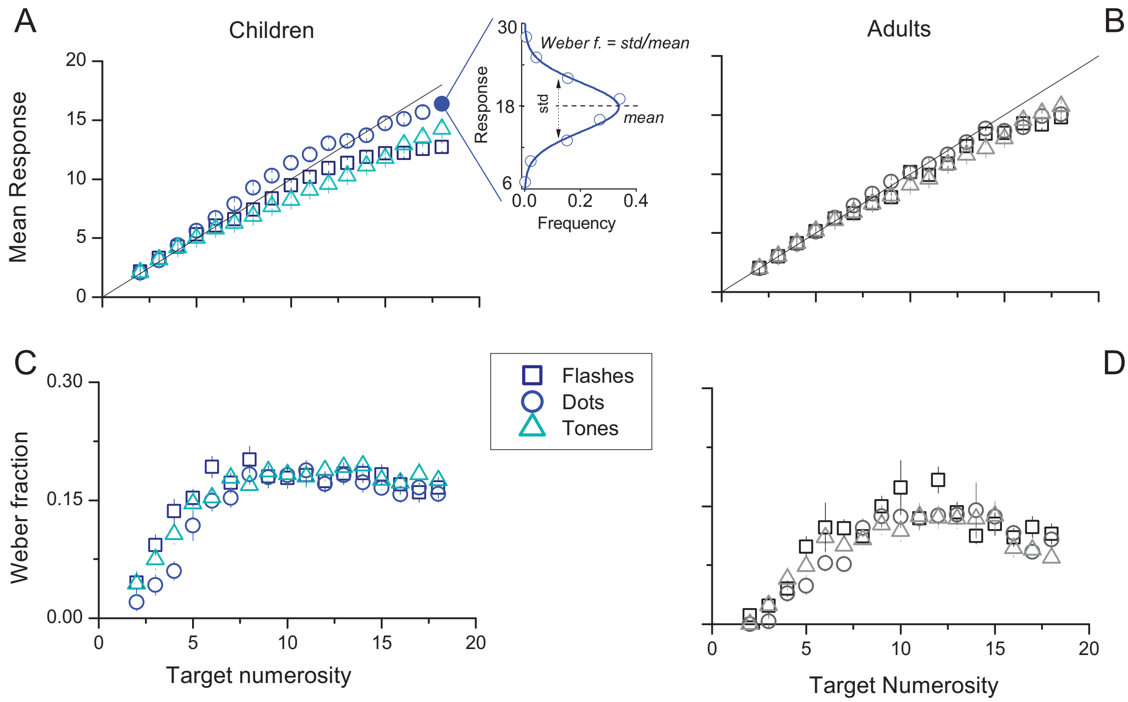

Figure 3. Psychophysical magnitude estimation tasks. (A, B) Average numerosity estimates as a function of target numerosity for three different kinds of stimuli: dot ensembles (circles), sequences of flashes (squares) or sounds (triangles). (C, D) Averaged Weber fractions, calculated by dividing the standard deviation of responses for each target numerosity by the perceived numerosity (see insert). Error bars indicate standard errors of the mean. See the online article for the color version of this figure.

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Task Reliability

Before testing for correlations between numerical precision in the psychophysical tasks and math skills, we ensured that reliability within the different tasks was similar. Reliability was measured separately for each task and group with a split-half method modified for psychophysical procedures and Cronbach's alpha (see Method for details). Reliability values are reported in Table 1 and were not statistically different either within or between groups (bootstrap sign test on split-half indexes, all $p > .05$).

Correlations Between Different Measures of ANS

We first looked for correlations in ANS precision within the four different psychophysical tasks in both primary schoolchildren and adults. We considered the paired-comparison task ("which has more?") and three different versions of the estimation task ("how many?") for dot ensembles, sequences of flashes or sequences of tones. If ANS generalizes across senses, stimulus presentation format (sequential or simultaneous) and task paradigm (estimation or paired-comparison), precision in all the ANS tasks should be
T2, 3 strongly correlated. Tables 2 and 3 report correlation coefficients, associated p values and Bayes factors (LBF) for children and adults respectively. Positive r values indicate that high precision in a given ANS tasks correspond to higher precision in another). Results with children (see Table 2) indicate that even when the effect of age and nonverbal IQ was controlled for (below diagonal), all the WFs measured by estimation tasks correlated positively and significantly with each other (simultaneous visual vs. sequential auditory $\rho_p = 0.41$; simultaneous visual vs. sequential visual $\rho_p = 0.32$; sequential visual vs. sequential auditory $\rho_p = 0.57$; all $p < .001$). Figure 5A–C shows a graphical representation F5
of children correlations (zero-order) for all combinations of magnitude estimation tasks. The pattern of results with adult participants (see Table 3) reveals a general trend similar to that found in children, with all estimation thresholds positively correlated with each other. However, only the correlation between sequential visual and sequential auditory estimation passed Bonferroni correction after the effect of nonverbal IQ was controlled for ($\rho_p = 0.57$, $p < .001$, alpha level: $0.05/15 = 0.0033$). These results show that performance in all the magnitude estimation tasks were positively correlated with each other, suggesting a common mechanism.

Correlations Between Paradigms

Here we asked whether ANS generalizes across paradigms: between magnitude estimation and paired-comparison (Figure 5D–F). In children, WFs measured with the paired-comparison task correlate positively with those measured with the magnitude estimation paradigms, but no correlation reached the statistical significance level (Bonferroni corrected $\alpha = 0.003$), although two out three correlations were very close ($r = .266$, $r = .262$, for tones and spatial arrays respectively, both $p = .004$). Because of the conservative Bonferroni corrected alpha level of $p < .003$, we also checked for the relationship between magnitude estimation tasks

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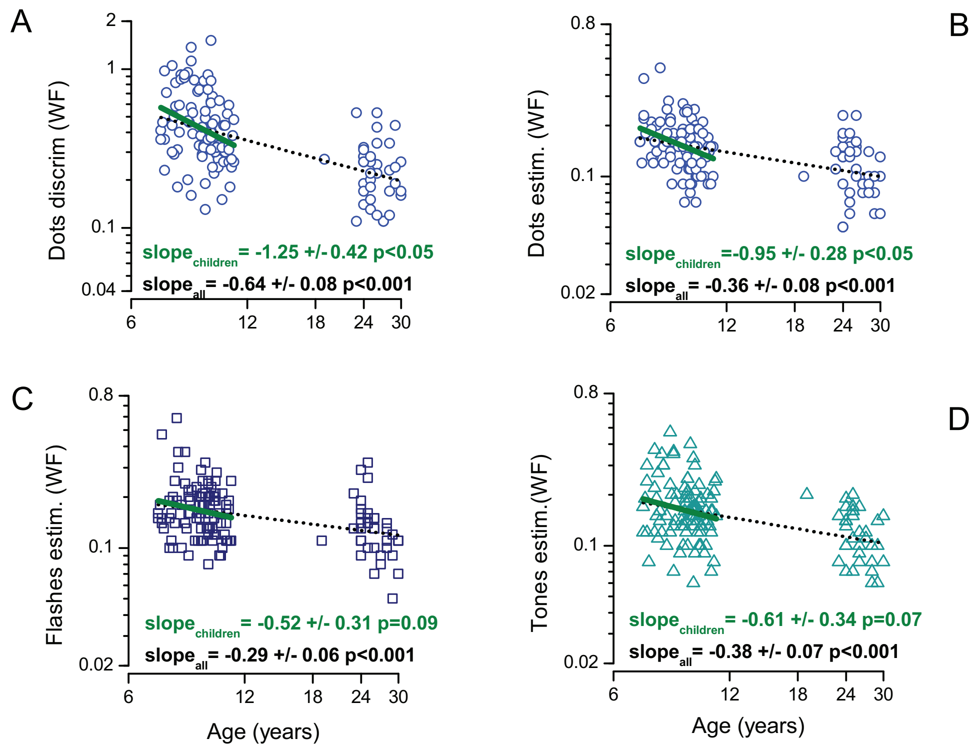

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Figure 4. Approximate number system developmental trajectories. Individual thresholds in the four approximate number system tasks as a function of chronological age: (A) spatial discrimination, (B) spatial estimation, (C) temporal estimation of flashes, and (D) temporal estimation of sounds. Black (dotted) lines are best-fitting regressions when both children and adults were considered, green (continuous) lines when only children were taken into account. See the online article for the color version of this figure.

and spatial discrimination using a regression model. We used spatial discrimination thresholds as dependent variable and all the magnitude estimation thresholds together as predictors. This analysis confirmed the independence between performance on magnitude estimation tasks and spatial discrimination (flashes: $\beta = -0.16$, $t = -1.23$ $p = .22$; dots spatial arrays: $\beta = 0.14$, $t = 1.16$, $p = .247$; tones: $\beta = 0.14$, $t = 2.02$, $p = .05$). For adult participants (see Table 3), all correlations were far from the significance level (the lowest was $p = .10$ for the correlation between estimation and paired-comparison of clouds of dots).

Approximate Number System and Math Abilities

We measured formal math by means of two tasks indexing semantic skills and mental calculation abilities. In the semantic tasks, children were required to quickly select the highest digit number between three options, or to place a target number between others, depending on their numerical magnitude (see Method for details). Mental calculation was measured by a "symbolic addition discrimination task" in which participants were required to rapidly mentally add two digit numbers (ranging from 1 to 9) and compare the result (ranging from

Table 2
Full Correlation Matrix: Children

Task	1	2	3	4	5	6
1. Math composite index	1	**−.29 (.002) [.68]**[a]	**−.30 (.002) [.8]**[a]	**−.12 (.122) [−.8]**[c]	**−.05 (.309) [−1]**[c]	**.07 (.238) [−.99]**
2. Numerosity discrimination (dots)	**−.26 (.007)**	1	**.26 (.004) [.36]**	**.26 (.004) [.34]**	**.09 (.172) [−.93]**	**.15 (.07) [−.62]**
3. Spatial estimation	**−.34 (<.001)**[a]	**.22 (.017)**	1	**.49 (<.001) [4.5]**[b]	**.48 (<.001) [4.3]**[b]	**.26 .005 [.34]**
4. Temporal tones estimation	−.09 (.194)	.18 (.035)	**.41 (<.001)**[b]	1	**.48 (<.001) [4.3]**[b]	**.48 (<.001) [4.3]**[b]
5. Temporal flash estimation	.14 (.083)	.05 (.308)	**.32 (<.001)**[b]	**.57 (<.001)**[b]	1	.26 .005 [.34]
6. Size discrimination	.19 (.031)	.08 (.213)	.09 (.1954)	**.33 (<.001)**[b]	**.23 (.01)**	1

AQ: 19 *Note*. Above diagonal: zero-order Pearson *r* coefficients; below diagonal: partial Spearman rho coefficients (age and nonverbal reasoning controlled). Significant correlations are highlighted in bold. One-tailed *p* values are reported in parentheses; log10 Bayes factors are reported in square brackets. Alpha level Bonferroni corrected = .0033 (.05/15 comparison).
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[a] Significant correlations between math skills and numerosities. [b] Significant correlations between different numerosity estimates. [c] Correlations where the log10 Bayes factor suggests there exists strong evidence for zero correlation.

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Table 3
Full Correlation Matrix: Adults Above diagonal: Zero-Order Pearson R Coefficients Below diagonal: Partial Spearman Rho Coefficients (Non-Verbal Reasoning Controlled)

Task	1	2	3	4	5	6
1. Math (addition)	1	−.16 (.157) [−.7]	−.36 (.012) [.18]	−.32 (.02) [−.05]	−.26 (.05) [−.35]	−.44 (.003) [.78]
2. Numerosity discrimination (dots)	−.02 (.449)	1	.21 (.096) [−.54]	.16 (.156) [−.7]	.05 (.373) [−.88]	.27 (.046) [−.3]
3. Spatial estimation	−.33 (.021)	.178 (.144)	1	.22 (.08) [−.5]	**.46 (.001) [.96]**	**.53 (<.001) [1.68]**
4. Temporal tones estimation	−.33 (.021)	.16 (.166)	.06 (.171)	1	**.57 (<.001) [2.1]**	.40 (.013) [.46]
5. Temporal flash estimation	−.33 (.021)	−.003 (.493)	.387 (.004)	**.57 (<.001)**	1	.36 (.012) [.18]
6. Size discrimination	−.16 (.161)	.27 (.049)	**.42 (.002)**	.43 (.003)	.30 (.03)	1

Note. Above diagonal: zero-order Pearson *r* coefficients; below diagonal: partial Spearman rho coefficients (nonverbal reasoning controlled). Significant correlations are highlighted in bold. One-tailed *p* values are reported in parentheses; log Bayes factors are reported in square brackets. Alpha level Bonferroni corrected = .0033 (.05/15 comparison).

5 to 10) with a given comparison number. Negative correlations indicate that higher numerosity precision correspond to higher math abilities.

Children

Semantic tasks and mental addition were correlated between each other ($r = .42$, $p < .001$) and both correlate well with precision in estimation ($r = -0.27$, $p = .003$; $r = -0.23$, $p = .01$, for semantic and addition tasks respectively) and discrimination of simultaneous visual numerosity ($r = -0.30$, $p = .001$; $r = -0.21$, $p = .01$, for semantic and addition tasks, respectively). Importantly for the purpose of this study, precision in sequential numerosity (sequences of flashes or tones) was unrelated to both math tasks (flashes vs. semantic task $r = -0.01$, $p = .44$; flashes vs. addition $r = -0.1$, $p = .163$; tones vs. semantic task $r = -0.06$, $p = .25$; flashes vs. addition $r = -0.15$, $p = .07$) even when no covariates were controlled for. Given that both mathematical tasks were correlated and similarly related to numerosity performance, to gain information reducing the number of variables, we built a summary math index (math composite index) by averaging the *z* scores for
F6 the semantic and calculation task. Figure 6 reports zero-order correlations between math composite index and the four different measures of the ANS. As before, only simultaneous visual numerosity was found to significantly correlate with math abilities (Figure 6A and 6B), while even with this liberal analysis (no covariates were controlled for) sequential numerosity was not related to math skills (Figure 6C and 6D for flashes and tones estimations respectively). When controlling for age and nonverbal reasoning, both simultaneous numerosity tasks were still related to math ($\rho_p = -0.26$, $p = .007$; $\rho_p = -0.34$, $p < .001$ for paired-comparison and estimation), but only the correlation for estimation survived the Bonferroni correction ($\alpha = .0033$).

The correlational approach used so far may risk being too conservative, as the high number of comparison variables (15) led to a very conservative alpha level ($\alpha = .003$), and it is not very informative regarding the real extent of explained variance. We therefore performed a series of hierarchical regression analyses with the numerosity thresholds as predictors and the math composite index as the dependent variable. Each predictor was tested in a separate model and the controlling variables were entered each time together as a block. Age and nonverbal reasoning, together, explain 10% of math variance, $F_{change}(92) = 5.25$, $p = .007$ (see
T4 Table 4). The two simultaneous visual numerosity thresholds explained an additional significant 10.2% of variance, $F_{change}(90) = 5.8$, $p = .004$, almost equally distributed between estimation and discrimination (6% and 5.6% respectively, see Table 4). We then asked whether simultaneous estimation and paired-comparison thresholds contribute to math independently, or whether their contributions are shared. We performed two separate hierarchical regression analyses, with one of the two spatial numerosity thresholds as predictor and the math composite index as the dependent variable. Crucially, the controlling variables this time included age, nonverbal reasoning and also the simultaneous numerosity thresholds not used as predictors. Table 5 shows that both spatial T5
thresholds contributed independently to math, explaining about 4% of variance each.

To rule out the possibility that the lack of correlation between math and sequential numerosity magnitude estimation was due to differences in intersubject variability between simultaneous and sequential numerosity tasks, we ran a series of bootstrap sign-tests on task variance ratio (10,000 iteration, sample-with-replacement). On each iteration and for each condition, we computed the group WF variance and their ratios (dots/flashes and dots/sounds). The *p* values were derived by the proportion of times the ratio values were higher than 1, implying higher dots WF variance. The *p* values were 0.49 for the comparison between simultaneous numerosity and sequential visual numerosity, and 0.48 between simultaneous and sequential auditory numerosity (see also Table 1 for groups task standard deviation), indicating that these tasks had similar variability levels. Another possibility is that sequential numerosity judgments do not involve approximate estimates. We controlled for this by taking advantage of the fact that the main feature of ANS is that it obeys Weber Law: variability linearly scales with numerosity, leaving WF (variability/numerosity) stable across numerosities (Anobile et al., 2014; Anobile, Turi, Cicchini, & Burr, 2015; Dehaene, 2011; Ross, 2003). Thus, for each condition we tested whether estimation obeys Weber law. Separately for each estimation tasks and for each numerosity (5–16), we measured the between average WF (see Figure 3C) and fitted it with a linear regression model: a slope of zero implies that the WF is constant across all tested numerosities. All the measured slopes were not different from zero, suggesting that numerosity estimates obeyed Weber's law in all conditions (spatial: slope = 0.002 ± 0.002, $p = .15$; flashes: slope = -0.004 ± 0.008, $p = .292$; sounds: slope = 0.0016 ± 0.004, $p = .069$).

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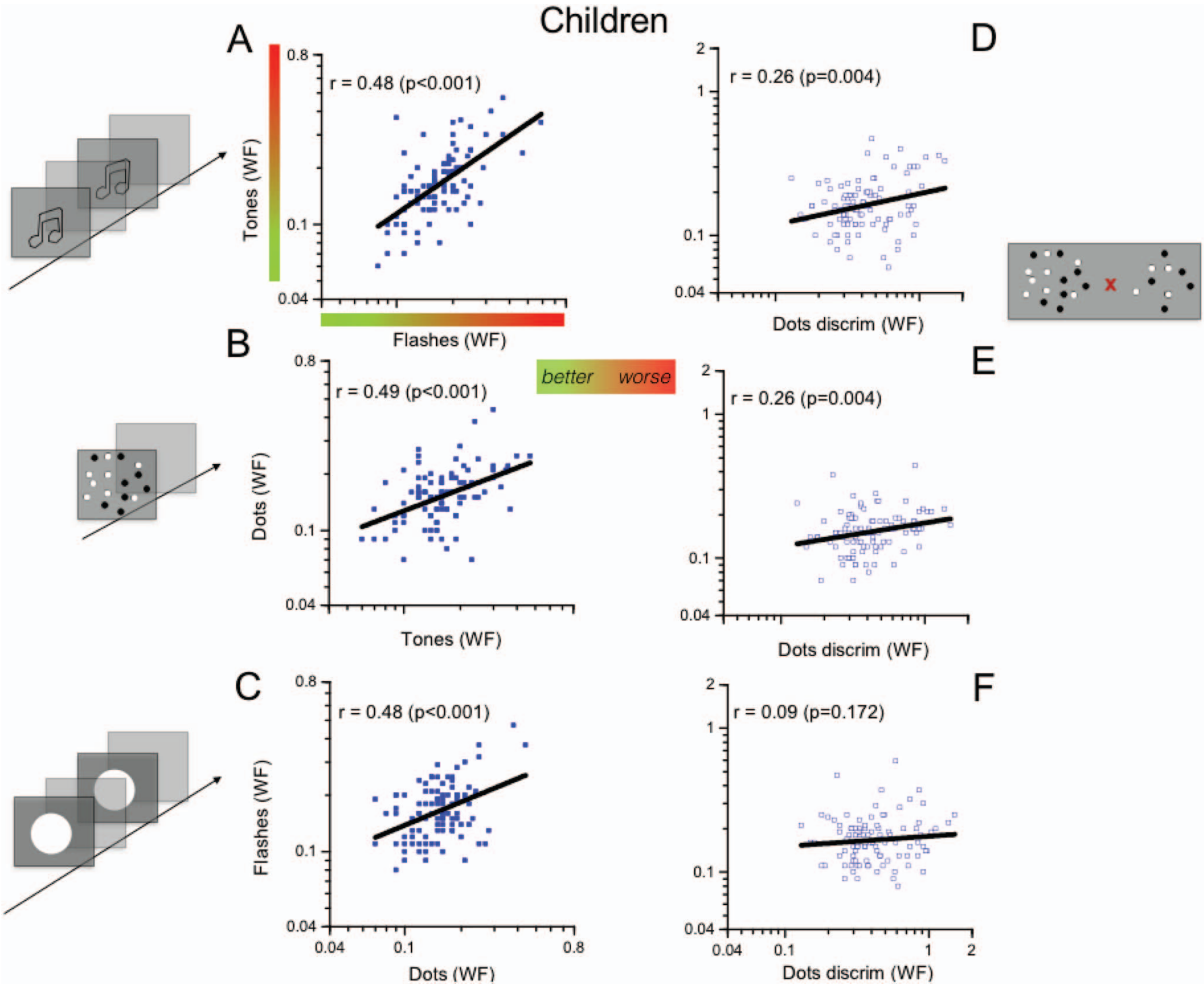

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Figure 5. Correlations between approximate number system (ANS) precision measures in children. (A–C) Correlations between different ANS measures obtained with magnitude estimation tasks ("how many?"). (D–F) Correlations between ANS measures obtained by magnitude estimation tasks against that obtained by a spatial (dots) numerosity discrimination (two-alternative forced choice, "which most numerous?") task. Filled symbols report statistically significant correlations (Pearson zero-order correlations with alpha-level: 0.05/15 comparisons = 0.003; see Table 2 for partial Spearman rho correlations). See the online article for the color version of this figure.

Adults. Adult math skills were tested only with the mental calculation task as the semantic task was designed for children and would saturate with adults. Table 3 shows that adult math skills were not related to any of the ANS measures, even when no covariates were controlled for. As for children, the correlational approach risks being too conservative because the high number of comparison (corrected alpha level of 0.003). We therefore also performed hierarchical regressions to measure the how much math variance can be explained by the single ANS components. As the nonverbal reasoning by itself explains a significant portion of math variance (15.7%), $F(1,36) = 6.718$, $p = .01$, we always entered it as control variable each time in the first step. Table 6 shows that none of the numerosity tasks explains a significant portion of math variance (all $p > .05$). We then tried a more liberal analysis, performing a regression analyses with math as dependent variables and all the numerical thresholds together—as a block—as predictor and no controlling variables. With this analysis, where all the numerosity tasks can sum together their contribution, we found that together they explained 21% of math variance, still not sufficient to reach the statistical significance level, $F(4,33) = 2.19$, $p = .09$.

T6

One possible explanation for the lack of correlation might be that math abilities are too similar to each other so there is not enough intersubject variability to drive significant covariance. To test for this, we ran a bootstrap analysis on math abilities variance between adults and children. On each iteration (10,000

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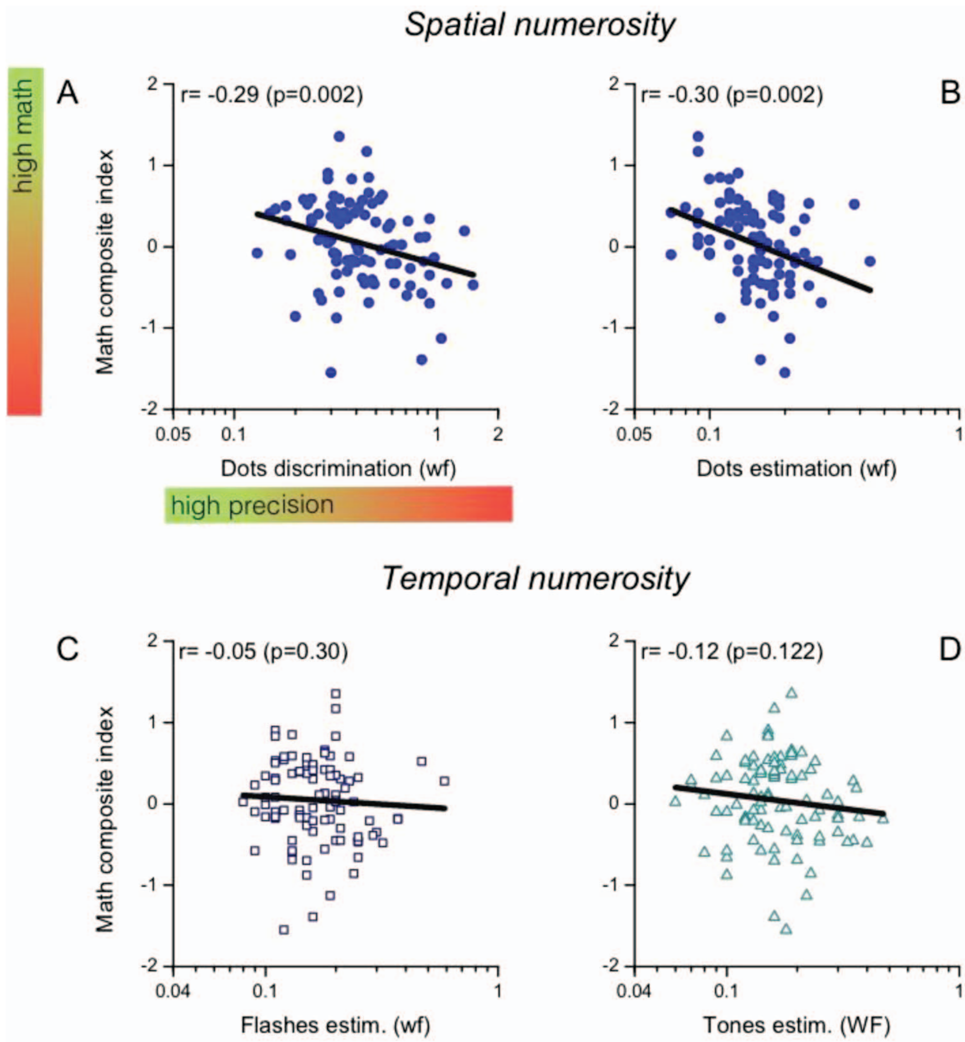

Figure 6. Correlations between approximate number system (ANS) and math skills in children. ANS Weber fractions are plotted against standardized math skills level for the four ANS tasks: (A) spatial ensemble discrimination, (B) spatial estimation, (C) estimation of flashes sequences, and (D) estimation of tones sequences. Filled symbols report statistically significant correlations (Pearson zero-order correlations with alpha-level = 0.05/15 = 0.0033; see Table 2 for partial Spearman rho correlations). See the online article for the color version of this figure.

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in total), we sampled (with replacement) the data in the two groups, and computed the ratio of the two variances. We then counted the proportion of trials where the ratio was higher than 1, giving the probability (one-tailed *p* value) that the adults had less variance than children. The *p* value was 0.028, confirming that the adult group had less variance in math scores (mental calculation in this case) than children, leaving open the possibility that the lack of correlation between simultaneous visual numerosity and math in adults might be simply explained in these terms.

Correlations Between Size-Discrimination and Math

The results described above suggest that the best predictors of formal math skills in children are ANS sensitivities (inverse WF) for simultaneous visual numerosity (ensembles of dots). However, it is not clear whether the correlation between simultaneous visual numerosity sensitivity and math is driven by the visual comparison process itself (in this case the decision-related process of the discrimination task that might have triggered per se the correlation), or whether these correlations are specific for simultaneous numerosity processing. We therefore also measured sensitivity on a suitable control—size-discrimination—a visual task thought not to require processing of numerosity: participants reported which of two disks was larger (see methods for details). The results show that size discrimination thresholds were unrelated to math skills in children (see Table 2). This control suggests that the correlation between simultaneous numerosity sensitivity and math ability is not driven by the comparison process itself, but is specific for

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Table 4
Contribution of Approximate Number System Components on Children Math

Model	Predictor	R^2	R^2_{change}	F_{change}	*df*	*p*
First step	Age and nonverbal IQ	.101	—	5.153	92	.007*
Spatial (dots)						
Model 1	Paired comparison	**.156**	**5.6%**	**6.061**	**92**	**.016***
Model 2	Estimation	**.161**	**6%**	**6.526**	**91**	**.012***
Temporal (sequences)						
Model 3	Flashes	.102	.1%	.137	92	.712
Model 4	Tones	.106	.5%	.524	92	.471

Note. Hierarchical multiple regressions—dependent variable: mathematical composite index; Controlling variables: age and nonverbal IQ. Predictors were tested in separate regressions models (Models 1, 2, 3, and 4); controlling variables were entered as a block in the first step. Significant predictors are highlighted in bold.
* $p < .05$.

numerosity encoding. We also looked for possible correlations between math and size discrimination in adults. Although zero-order correlation reveals a slightly significant correlation ($r = -0.44$, $p = .003$, $\alpha = .003$), when nonverbal reasoning skills were controlled for, this correlation become insignificant ($\rho_p = -0.16$, $p = .16$, see Table 3). As above, we also controlled for lack or difference of variability and precision (WF) between size and numerosity tasks. Bootstrap analysis on both adults and children revealed that size discrimination had lower variability and higher precision compared with both numerosity discrimination and estimation (all $p < .01$).

Discussion

This study aimed to measure the relationship between ANS sensitivity and math abilities by investigating the role of stimulus sensory modality (vision or audition), presentation format (simultaneous or sequential), task requirements (paired comparisons or verbal estimation), and magnitude dimensions (numerosity or size), in both children and adults.

We first replicated previous studies showing that children with higher precision in estimating and discriminating simultaneous visual numerosity show higher abilities in formal math (Anobile et al., 2013; Chen & Li, 2014; Cicchini et al., 2016; Feigenson, Libertus, & Halberda, 2013; Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2013; Piazza et al., 2010). However, we also found that precision in estimating sequential numerosity—sequences of flashes or sounds—was completely unrelated to math abilities, both in children and in adults. Performance on a control nonnumerical discrimination task (stimuli size) was also unrelated to math abilities, showing that the correlation between simultaneous numerosity discrimination and math was not unspecifically driven by the visual discrimination processes itself. Furthermore, the lack of correlation in children between sequential numerosity and math cannot be accounted for by trivial methodological issues, as the data in all magnitude estimation tasks obeyed Weber Law, were correlated with each other, and had similar reliability levels and intersubject variability. We also quantified the strength of these null correlations by means of LBF. LBFs for correlations between sequential numerosity and math skill were both near −1 (Table 2, red cells), indicating very strong evidence in favor of the null hypothesis of zero correlation (Wetzels & Wagenmakers, 2012).

On the other hand, we believe these results suggest that math reasoning has a specific relationship with the encoding of spatial information about quantity. This relationship, however, may diminish in adulthood, as adult ANS acuity for spatial numerosity estimation and discrimination did not correlate with simple math

Table 5
Separate Contribution of Spatial Numerosity Thresholds on Children Math

Predictor	R^2	R^2_{change}	F_{change}	*df*	*p*
Controlling variables: Age, nonverbal IQ, and dots paired comparison					
First step					
Age and nonverbal IQ and dots paired comparison	.156	—	5.620	91	.001
Model 1A					
Spatial estimation	**.202**	**4.5%**	**5.124**	**90**	**.026***
Controlling variables: Age, nonverbal IQ, and spatial numerosity estimation					
First step					
Age and nonverbal IQ and Spatial estimation	.161	—	5.817	91	.001
Model 1B					
Dots paired-comparison	**.202**	**4.1%**	**4.607**	**90**	**.035***

Note. Hierarchical multiple regressions—dependent variable: mathematical composite index. Predictors were tested in separate regressions models (Model 1A, 1B); controlling variables were entered as a block in the first step. Significant predictor highlighted in bold.
* $p < .05$.

Table 6
Contribution of Approximate Number System Components on Adult's Math

Model	Predictor	R^2	R^2_{change}	F_{change}	*df*	*p*
First step	Nonverbal IQ	.157	—	6.718	36	.014*
Spatial (dots)		.157	0%	.001	35	.972
Model 1	Discrimination	.2	4.3%	1.891	35	.178
Model 2	Estimation	.157	—	6.718	36	.014*
Temporal (sequences)						
Model 3	Flashes	.18	2.3%	.966	35	.333
Model 4	Tones	.2	4.2%	1.856	35	.182

Note. Hierarchical multiple regressions—dependent variable: mathematical composite index; Controlling variable: nonverbal IQ. Predictors were tested in separate regressions models (Models 1, 2, 3, and 4); controlling variables were entered as a block in the first step. Significant predictors are highlighted in bold.
* $p < .05$.

performance. This last result suggests that ANS may act as a start-up tool for math achievements until symbolic quantity are precisely mapped onto nonsymbolic quantities; but later on the two processes may become independent. However, given that adult math skills had less intersubject variability than children, we cannot completely rule out the possibility that the lack of correlation was simply due to an insufficient variability level in one of the dimensions. In addition, we had only one test of math skills for the adults, which may have been insufficient. Previous studies have reported both nonsignificant correlations (Inglis, Attridge, Batchelor, & Gilmore, 2011; Krueger, 1984) and significant correlations (Chen & Li, 2014; Fazio et al., 2014; Libertus, Odic, & Halberda, 2012; Schneider et al., 2017; Tibber et al., 2013) between spatial processing and math abilities in adulthood. There is also considerable variability in the correlations observed in studies on children (Anobile et al., 2013; Libertus, Odic, Feigenson, & Halberda, 2016; Piazza et al., 2010). Some of the discrepancies in these findings may be explained by the different tests used to assess formal math abilities (Lourenco, Bonny, Fernandez, & Rao, 2012). For this reason in the present study we used the same tests (mental addition) to assess math capacities in both children and adults. Furthermore, Bonny and Lourenco (2013) found a clear link between math and ANS acuity only in lower math preschoolers, with far less evidence for this relation among the higher performers. However, Wang, Halberda, and Feigenson (2017) found robust correlations when considering only math-gifted adolescents, suggesting that the correlation is robust even in high performers. There is clearly a good deal of inconsistency in the literature at this stage.

Why does spatial simultaneous visual numerosity, but not sequential numerosity perception correlate with child math abilities? Many works have highlighted the intimate relationship between spatial reasoning and math abilities, leading to the hypothesis that numbers are represented as spatially organized along a "mental numberline" (Dehaene, 2011; Dehaene & Brannon, 2011; Dehaene, Izard, Spelke, & Pica, 2008; Galton, 1880; Hubbard, Piazza, Pinel, & Dehaene, 2005). Correlational studies demonstrated that higher accuracy levels for mapping numbers onto spatial position (numberline tasks) but also nonnumerical spatial reasoning abilities are associated with higher formal math skills (Ashkenazi & Henik, 2010; Booth & Siegler, 2006; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Gunderson, Ramirez, Beilock, & Levine, 2012; Siegler & Booth, 2004; Siegler & Opfer, 2003). Remarkably, the link between visuospatial processing and math abilities has been reported since the first year of life: visuospatial abilities of 10-month-old infants, indexed by preferential looking time at streams of objects containing spatially mirrored objects, predicts their formal math abilities 3 years later (Lauer & Lourenco, 2016).

Basic numerosity perception, as well as many other visuospatial abilities, relies on brain regions that are also activated by math tasks. These areas—mainly frontoparietal—are also involved in time processing, space perception, geometrical relationships, visuospatial analogies, and are also activated by the mere sight of numbers and mathematical formulas, simple mental calculation, and high-level abstract mathematical reflection (Amalric & Dehaene, 2016; Dehaene, 2011; Watson & Chatterjee, 2012; Harvey, Fracasso, Petridou, & Dumoulin, 2015; Harvey, Klein, Petridou, & Dumoulin, 2013; Hubbard et al., 2005; Nieder, 2016; Piazza, Pinel, Le Bihan, & Dehaene, 2007; Pinel, Piazza, Le Bihan, & Dehaene, 2004). Moreover, skilled mathematicians, when engaged in difficult mathematical tasks, recruit additional visual areas at the expense of others, including reduced activation to faces (Amalric & Dehaene, 2016). The human-specific development of formal math may have been rooted and may have taken advantage of brain areas that were already processing those visuospatial features that lends to math analogies. In line with this, studies searching for the origin of math thinking found that language seems to have a less or different weight than visuospatial skills (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Pica, Lemer, Izard, & Dehaene, 2004; Dehaene et al., 2008; Dillon, Huang, & Spelke, 2013). More specifically, training approximate calculation (e.g., estimate the result and indicate the closest, or decide whether 8 is closer to 9 or 5) in bilingual participants does not suffer any language-shifting costs, as do exact calculations; and approximate calculation tasks activate the same brain areas (around the intraparietal sulcus) that are active during a variety of visuospatial tasks (Dehaene et al., 1999).

We recently replicated this pattern of results showing that approximate math but not exact calculation correlates with children's math abilities (Anobile et al., 2013). It is worth noting that also in the present study we measured approximate tasks in formal math, so the possibility remains open that sequential numerosity may be related to other kinds of math components. For example, as auditory sequences are important for language, and it has been recently reported that changes in auditory numerosity elicit activation in the

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same parietal areas activated by visual-spatial numerosity (Wang, Uhrig, Jarraya, & Dehaene, 2015); a speculative hypothesis is that auditory sequential numerosity sensitivity might be related to language-based arithmetic skills (e.g., Multiplication table).

How specific is the link of spatial simultaneous numerosity and math? Tibber et al. (2013) found that higher performance on a task requiring object orientation perception and reproduction was related to higher math skills, opening the possibility that math may be unspecifically related to higher visuospatial sensitivity. However, much evidence suggests that it may be not the case, indeed, sensitivity to visual object distance (density) was seen to be unrelated to math (Tibber et al., 2013). Piazza et al. (2013) showed that increase in formal math knowledge is associated with an increase in spatial numerosity discrimination sensitivity leaving unchanged the precision to discriminate object size. Anobile et al. (2013) showed that sensitivity to the numerosity of relatively sparse but not dense patterns of objects was correlated with math and visual motion direction sensitivity, was not correlated with math. Here we replicated Piazza et al. (2013) showing that object size discrimination does not correlate with formal math.

It is also important to note that size discrimination showed intersubject variability and higher levels of precision compared with both simultaneous numerosity tasks (for both children and adults, all $p < .01$). Even if this evidence should be considered as pointing to different developmental processes underling numerosity and size perception, it may also have obscured the correlation between size and math. However, object size discrimination is particularly suitable to serve as such a control, as it shares with numerosity many key perceptual and task-related features: it involves spatial visual stimulation, continuous magnitude encoding, an identical decision process ("which is larger"), a similar way of responding ("left–right"), and the same presentation format (rapid, peripheral and simultaneous). Furthermore, size and numerosity are both encoded in the parietal cortex (Castaldi, Aagten-Murphy, Tosetti, Burr, & Morrone, 2016; Harvey et al., 2013; Nieder, 2016; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004), and share highly overlapping representational areas (Harvey, Fracasso, Petridou, & Dumoulin, 2015; Pinel et al., 2004). Nevertheless, Lourenco and
AQ: 11 colleagues (Lourenco & Bonny, 2017; Lourenco et al., 2012) found that precision in cumulative area discrimination correlates with numerosity discrimination, geometry and math in both adults and 5-year-old children. These results suggest that analog magnitude and math achievement may correlate with nonnumerical dimensions such as cumulative area, even at the early stages of development, arguing against an exclusive role for nonsymbolic numbers in promoting math learning. While this is interesting, it should be noted that cumulative area is a task requiring ensemble perception, like numerosity. On the other hand, our area task did not require integration of multiple items, which may explain the apparently conflicting result. Moreover, recent evidence suggests that numerosity, area and other features (like density) develop independently of each other, with different developmental trajectories, and different links with other math capabilities (Odic et al., 2013; Anobile, Castaldi, Turi, Tinelli, & Burr, 2016). However, a comprehensive review of relationship between continuous and discrete quantities goes far beyond the scope of this study. We refer interested readers to Leibovich et al. (2017) and commentaries on that article for further discussion.

. The results show that while WFs for all the magnitude-estimation tasks positively correlated with each other in both children and adults, magnitude estimations correlate with paired comparisons to a lesser extent. More precisely, although children showed some degree of between-task generalization (precision in paired comparisons correlates with sensitivity for estimation of dot-cloud and tones numerosity), this pattern of results did not hold for adults. Taken together, these findings suggest that ANS may start as a highly generalized system, which subsequently segregates the various components of numerical processing during the development. The children tested here varied in age from 7 to 8 years, so the hypothesized specialization should take place during later developmental stages. Given that also in the adult group most of the intertask correlations were highly significant, and that both intratask reliability indexes and intersubjects variability were similar across perceptual tasks, we believe that the lack of correlation reflects a genuine lack or interrelationship.

As mentioned before, only paired comparison and estimation of simultaneous spatial arrays correlated with child math skills. How can this result be reconciled with the fact that children who are more precise at estimation of simultaneous stimuli also perform better with sequential numerosity tasks (sequences of flashes or sounds)? Our results suggest that two kinds of connections may link different symbolic and nonsymbolic math-related competences: one sensory (or decision) based, linking all the numerical magnitude estimation performances; and one math-specific, linking only spatial simultaneous numerosity (for all tasks) with symbolic math achievements (see Figure 7). Another (not mutually F7
exclusive) possibility is that the correlations between sequential

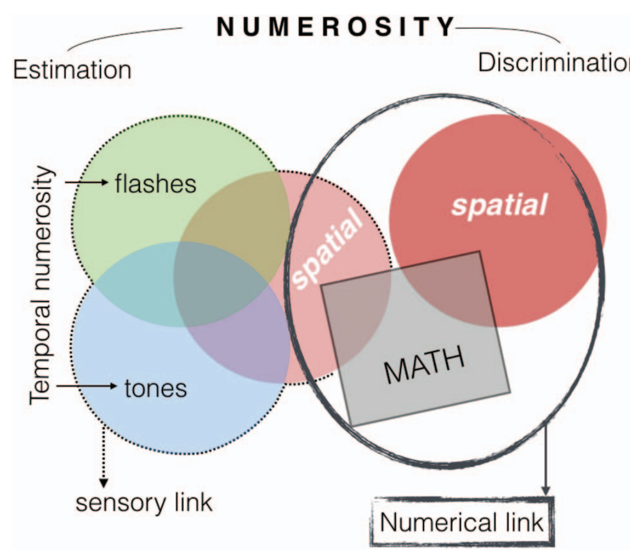

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Figure 7. Illustrative representation of the relationship between numerosity perception and children math skills. Nonsymbolic (numerosity) and symbolic (math) numerical abilities may be linked by two different correlations. On one side, a sensory link (dotted line) ties together the perception and estimation of numerosity regardless of its format (temporal or spatial) and sensory modality (visual or auditory). However, only sensory precision in spatial numerosity estimation and paired comparison relates to formal math abilities (gray circle line). We call this a "numerical link," as it gives a selective numerical meaning to spatial ensembles visual encoding. See the online article for the color version of this figure.

and simultaneous estimations were driven by shared verbal mapping processes transforming numerosity into number words. Unfortunately our battery of tests does not allow us to control for this covariate.

As this is the first study to measure the link between sequential numerosity and math, replications and extensions would be required before making very strong conclusions. However, our study confirms the idea that language-based human specific math abilities may have been built on top of a basic, ancient and generalized *numbersense*; but we add to this, showing that humans had developed a selective link between this symbolic system and visuospatial encoding of objects ensembles. Furthermore, this study opens new testable experimental questions: in congenitally blind children, who have not had visual experience, does the brain link sequential auditory numerosity and formal math? Another open question is how nonvisual spatial numerical encoding—such as tactile—relates to math.

Conclusions

The main finding of this study is that thresholds for judging the numerosity of temporal sequences do not relate to math skills, neither in children nor in adults, while child mathematical cognition correlates well with spatial (simultaneous) numerical encoding. These results are in line with the fascinating idea that human mathematical thought arises from the cultural recycling of ancient brain areas representing those basic features more naturally linked to math concepts, such as visual space (Dehaene, 2011; Dehaene et al., 1999; Dehaene & Cohen, 2007). Moreover, these results may be useful for teachers, clinicians and those who teach and reinforce formal mathematical learning using its nonsymbolic counterpart: numerosity. The present study, together with other evidence, suggests that to stimulate math achievement it may be more beneficial to rely on spatial, rather than temporal processing.

References

Amalric, M., & Dehaene, S. (2016). Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences of the United States of America, 113,* 4909–4917. http://dx.doi.org/10.1073/pnas.1603205113

Anobile, G., Arrighi, R., Togoli, I., & Burr, D. C. (2016). A shared numerical representation for action and perception. *eLife, 5,* e16161. http://dx.doi.org/10.7554/eLife.16161

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental Psychology, 52,* 1206–1216. http://dx.doi.org/10.1037/dev0000155

Anobile, G., Cicchini, G. M., & Burr, D. C. (2014). Separate mechanisms for perception of numerosity and density. *Psychological Science, 25,* 265–270. http://dx.doi.org/10.1177/0956797613501520

Anobile, G., Cicchini, G. M., & Burr, D. C. (2016). Number As a primary perceptual attribute: A review. *Perception, 45,* 5–31. http://dx.doi.org/10.1177/0301006615602599

Anobile, G., Stievano, P., & Burr, D. C. (2013). Visual sustained attention and numerosity sensitivity correlate with math achievement in children. *Journal of Experimental Child Psychology, 116,* 380–391. http://dx.doi.org/10.1016/j.jecp.2013.06.006

Anobile, G., Turi, M., Cicchini, G. M., & Burr, D. C. (2015). Mechanisms for perception of numerosity or texture-density are governed by crowding-like effects. *Journal of Vision, 15,* 4. http://dx.doi.org/10.1167/15.5.4

Arrighi, R., Togoli, I., & Burr, D. C. (2014). A generalized sense of number. *Proc Biol Sci, 281,* 20141797. http://dx.doi.org/10.1098/rspb.2014.1791

Ashkenazi, S., & Henik, A. (2010). A disassociation between physical and mental number bisection in developmental dyscalculia. *Neuropsychologia, 48,* 2861–2868. http://dx.doi.org/10.1016/j.neuropsychologia.2010.05.028

Barth, H., Kanwisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition, 86,* 201–221.

Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America, 102,* 14116–14121. http://dx.doi.org/10.1073/pnas.0505512102

Biancardi, A., Bachmann, C., & Nicoletti, C. (2016). *Batteria per la discalculia evolutiva (BDE2).* Trento, Italy: Centro studi erickson.

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology, 114,* 375–388. http://dx.doi.org/10.1016/j.jecp.2012.09.015

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42,* 189–201. http://dx.doi.org/10.1037/0012-1649.41.6.189

Brainard, D. H. (1997). The Psychophysics Toolbox. *Spatial Vision, 10,* 433–436. http://dx.doi.org/10.1163/156856897X00357

Brannon, E. M. (2003). Number knows no bounds. *Trends in Cognitive Sciences, 7,* 279–281. http://dx.doi.org/10.1016/S1364-6613(03)00137-2

Castaldi, E., Aagten-Murphy, D., Tosetti, M., Burr, D., & Morrone, M. C. (2016). Effects of adaptation on numerosity decoding in the human brain. *NeuroImage, 143,* 364–377. http://dx.doi.org/10.1016/j.neuroimage.2016.09.020

Chen, Q., & Li, J. (2014). Association between individual differences in nonsymbolic number acuity and math performance: A meta-analysis. *Acta Psychologica, 148,* 163–172. http://dx.doi.org/10.1016/j.actpsy.2014.01.016

Cicchini, G. M., Anobile, G., & Burr, D. C. (2016). Spontaneous perception of numerosity in humans. *Nature Communications, 7,* 12536. http://dx.doi.org/10.1038/ncomms12536

Dakin, S. C., Tibber, M. S., Greenwood, J. A., Kingdom, F. A. A., & Morgan, M. J. (2011). A common visual metric for approximate number and density. *Proceedings of the National Academy of Sciences of the United States of America, 108,* 19552–19557. http://dx.doi.org/10.1073/pnas.1113195108

Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. New York, NY: Oxford University Press.

Dehaene, S., & Brannon, E. (2011). *Space, time and number in the brain: Searching for the foundations of mathematical thought: An attention and performance series volume* (1st ed.). San Diego, CA: Elsevier Academic Press.

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron, 56,* 384–398. http://dx.doi.org/10.1016/j.neuron.2007.10.004

Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science, 311,* 381–384. http://dx.doi.org/10.1126/science.1121739

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science, 320,* 1217–1220. http://dx.doi.org/10.1126/science.1156540

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20,* 487–506. http://dx.doi.org/10.1080/02643290244000239

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284,* 970–974. http://dx.doi.org/10.1126/science.284.5416.970

Dillon, M. R., Huang, Y., & Spelke, E. S. (2013). Core foundations of abstract geometry. *Proceedings of the National Academy of Sciences of the United States of America, 110,* 14191–14195. http://dx.doi.org/10.1073/pnas.1312640110

Durgin, F. H. (2008). Texture density adaptation and visual number revisited. *Current Biology, 18,* R855–R856. http://dx.doi.org/10.1016/j.cub.2008.07.053

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123,* 53–72. http://dx.doi.org/10.1016/j.jecp.2014.01.013

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8,* 307–314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives, 7,* 74–79. http://dx.doi.org/10.1111/cdep.12019

Galton, F. (1880). Visualized numerals. *Nature, 21,* 252–256, 494–495.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78,* 1343–1359. http://dx.doi.org/10.1111/j.1467-8624.2007.01069.x

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33,* 277–299. http://dx.doi.org/10.1080/87565640801982361

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology, 48,* 1229–1241. http://dx.doi.org/10.1037/a0027433

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences of the United States of America, 109,* 11116–11120. http://dx.doi.org/10.1073/pnas.1200196109

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in nonverbal number acuity correlate with maths achievement. *Nature, 455,* 665–668. http://dx.doi.org/10.1038/nature07246

Harvey, B. M., Fracasso, A., Petridou, N., & Dumoulin, S. O. (2015). Topographic representations of object size and relationships with numerosity reveal generalized quantity processing in human parietal cortex. *Proceedings of the National Academy of Sciences of the United States of America, 112,* 13525–13530. http://dx.doi.org/10.1073/pnas.1515414112

Harvey, B. M., Klein, B. P., Petridou, N., & Dumoulin, S. O. (2013). Topographic representation of numerosity in the human parietal cortex. *Science, 341,* 1123–1126. http://dx.doi.org/10.1126/science.1239052

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6,* 435–448. http://dx.doi.org/10.1038/nrn1684

Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief nonsymbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition, 131,* 92–107. http://dx.doi.org/10.1016/j.cognition.2013.12.007

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America, 106,* 10382–10385.

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: But only in children. *Psychonomic Bulletin & Review, 18,* 1222–1229. http://dx.doi.org/10.3758/s13423-011-0154-1

Jordan, K. E., & Brannon, E. M. (2006). The multisensory representation of number in infancy. *Proceedings of the National Academy of Sciences of the United States of America, 103,* 3486–3489. http://dx.doi.org/10.1073/pnas.0508107103

Krueger, L. E. (1984). Perceived numerosity: A comparison of magnitude production, magnitude estimation, and discrimination judgments. *Perception & Psychophysics, 35,* 536–542. http://dx.doi.org/10.3758/BF03205949

Lauer, J. E., & Lourenco, S. F. (2016). Spatial processing in infancy predicts both spatial and mathematical aptitude in childhood. *Psychological Science, 27,* 1291–1298. http://dx.doi.org/10.1177/0956797616655977

Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From "sense of number" to "sense of magnitude": The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences, 40,* e164. http://dx.doi.org/10.1017/S0140525X16000960

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability? *Learning and Individual Differences, 25,* 126–133. http://dx.doi.org/10.1016/j.lindif.2013.02.001

Libertus, M. E., Odic, D., Feigenson, L., & Halberda, J. (2016). The precision of mapping between number words and the approximate number system predicts children's formal math abilities. *Journal of Experimental Child Psychology, 150,* 207–226. http://dx.doi.org/10.1016/j.jecp.2016.06.003

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica, 141,* 373–379. http://dx.doi.org/10.1016/j.actpsy.2012.09.009

Lindskog, M., & Winman, A. (2016). No evidence of learning in nonsymbolic numerical tasks: A comment on Park and Brannon (2014). *Cognition, 150,* 243–247. http://dx.doi.org/10.1016/j.cognition.2016.01.005

Lourenco, S. F., & Bonny, J. W. (2017). Representations of numerical and nonnumerical magnitude both contribute to mathematical competence in children. *Developmental Science, 20,* e12418. http://dx.doi.org/10.1111/desc.12418

Lourenco, S. F., Bonny, J. W., Fernandez, E. P., & Rao, S. (2012). Nonsymbolic number and cumulative area representations contribute shared and unique variance to symbolic math competence. *Proceedings of the National Academy of Sciences of the United States of America, 109,* 18737–18742. http://dx.doi.org/10.1073/pnas.1207212109

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science, 17,* 714–726. http://dx.doi.org/10.1111/desc.12152

Nieder, A. (2016). The neuronal code for number. *Nature Reviews Neuroscience, 17,* 366–382. http://dx.doi.org/10.1038/nrn.2016.40

Obersteiner, A., Reiss, K., & Ufer, S. (2013). How training on exact or approximate mental representations of number can enhance first-grade students' basic number processing and arithmetic skills. *Learning and Instruction, 23,* 125–135. http://dx.doi.org/10.1016/j.learninstruc.2012.08.004

Odic, D., Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Developmental change in the acuity of approximate number and area representations. *Developmental Psychology, 49,* 1103–1112. http://dx.doi.org/10.1037/a0029472

Park, J., Bermudez, V., Roberts, R. C., & Brannon, E. M. (2016). Non-symbolic approximate arithmetic training improves math performance in preschoolers. *Journal of Experimental Child Psychology, 152,* 278–293. http://dx.doi.org/10.1016/j.jecp.2016.07.011

Park, J., & Brannon, E. M. (2013). Training the approximate number system improves math proficiency. *Psychological Science, 24,* 2013–2019. http://dx.doi.org/10.1177/0956797613482944

Park, J., & Brannon, E. M. (2014). Improving arithmetic performance with number sense training: An investigation of underlying mechanism. *Cognition, 133,* 188–200. http://dx.doi.org/10.1016/j.cognition.2014.06.011

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences, 14,* 542–551. http://dx.doi.org/10.1016/j.tics.2010.09.008

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., . . . Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition, 116,* 33–41. http://dx.doi.org/10.1016/j.cognition.2010.03.012

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron, 44,* 547–555. http://dx.doi.org/10.1016/j.neuron.2004.10.014

Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal approximate number system. *Psychological Science, 24,* 1037–1043. http://dx.doi.org/10.1177/0956797612464057

Piazza, M., Pinel, P., Le Bihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron, 53,* 293–305. http://dx.doi.org/10.1016/j.neuron.2006.11.022

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306,* 499–503. http://dx.doi.org/10.1126/science.1102085

Pinel, P., Piazza, M., Le Bihan, D., & Dehaene, S. (2004). Distributed and overlapping cerebral representations of number, size, and luminance during comparative judgments. *Neuron, 41,* 983–993. http://dx.doi.org/10.1016/S0896-6273(04)00107-2

Pooresmaeili, A., Arrighi, R., Biagi, L., & Morrone, M. C. (2013). Blood oxygen level-dependent activation of the primary visual cortex predicts size adaptation illusion. *The Journal of Neuroscience, 33,* 15999–16008. http://dx.doi.org/10.1523/JNEUROSCI.1770-13.2013

Rasanen, P., Salminen, J., Wilson, A. J., Aunio, P., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. *Cognitive Development, 24,* 450–472. http://dx.doi.org/10.1016/j.cogdev.2009.09.003

Ross, J. (2003). Visual discrimination of number without counting. *Perception, 32,* 867–870. http://dx.doi.org/10.1068/p5029

Rousselle, L., & Noël, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs nonsymbolic number magnitude processing. *Cognition, 102,* 361–395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Sasanguie, D., Defever, E., Maertens, B., & Reynvoet, B. (2014). The approximate number system is not predictive for symbolic number processing in kindergarteners. *The Quarterly Journal of Experimental Psychology, 67,* 271–280. http://dx.doi.org/10.1080/17470218.2013.803581

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology, 30,* 344–357. http://dx.doi.org/10.1111/j.2044-835X.2011.02048.x

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of nonsymbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20,* e12372. http://dx.doi.org/10.1111/desc.12372

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75,* 428–444. http://dx.doi.org/10.1111/j.1467-8624.2004.00684.x

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14,* 237–243. http://dx.doi.org/10.1111/1467-9280.02438

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America, 110,* 18116–18120. http://dx.doi.org/10.1073/pnas.1302751110

Sullivan, J., Frank, M. C., & Barner, D. (2016). Intensive math training does not affect approximate number acuity: Evidence from a three-year longitudinal curriculum intervention. *Journal of Numerical Cognition, 2,* 57–76. http://dx.doi.org/10.5964/jnc.v2i2.19

Tibber, M. S., Manasseh, G. S. L., Clarke, R. C., Gagin, G., Swanbeck, S. N., Butterworth, B., . . . Dakin, S. C. (2013). Sensitivity to numerosity is not a unique visuospatial psychophysical predictor of mathematical ability. *Vision Research, 89,* 1–9. http://dx.doi.org/10.1016/j.visres.2013.06.006

Wang, J. J., Halberda, J., & Feigenson, L. (2017). Approximate number sense correlates with math performance in gifted adolescents. *Acta Psychologica, 176,* 78–84. http://dx.doi.org/10.1016/j.actpsy.2017.03.014

Wang, L., Uhrig, L., Jarraya, B., & Dehaene, S. (2015). Representation of numerical and sequential patterns in macaque and human brains. *Current Biology, 25,* 1966–1974. http://dx.doi.org/10.1016/j.cub.2015.06.035

Watson, A. B., & Pelli, D. G. (1983). QUEST: A Bayesian adaptive psychometric method. *Perception & Psychophysics, 33,* 113–120. http://dx.doi.org/10.3758/BF03202828

Watson, C. E., & Chatterjee, A. (2012). A bilateral frontoparietal network underlies visuospatial analogical reasoning. *NeuroImage, 59,* 2831–2838. http://dx.doi.org/10.1016/j.neuroimage.2011.09.030

Wetzels, R., & Wagenmakers, E. J. (2012). A default Bayesian hypothesis test for correlations and partial correlations. *Psychonomic Bulletin & Review, 19,* 1057–1064. http://dx.doi.org/10.3758/s13423-012-0295-x

Wilson, A. J., Dehaene, S., Dubois, O., & Fayol, M. (2009). Effects of an adaptive game intervention on accessing number sense in low-socioeconomic-status kindergarten children. *Mind, Brain and Education, 3,* 224–234. http://dx.doi.org/10.1111/j.1751-228X.2009.01075.x

Wilson, A. J., Dehaene, S., Pinel, P., Revkin, S. K., Cohen, L., & Cohen, D. (2006). Principles underlying the design of "The Number Race", an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions, 2,* 19. http://dx.doi.org/10.1186/1744-9081-2-19

Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of "The Number Race", an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions, 2,* 20. http://dx.doi.org/10.1186/1744-9081-2-20

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